Facts that Matter

- An algebraic expression, in which the variables involved have only whole number powers, is called a polynomial.
- Polynomials of degree 1, 2 and 3 are called linear, quadratic and cubic polynomials respectively.
- Polynomials containing one, two and three non-zero terms are called monomial, binomial and trinomial respectively.
- If \( f(x) \) be a polynomial of degree \( n \geq 1 \) and let ‘\( a \)’ be any real number, then \( f(a) \) is the remainder for ‘\( f(x) \) being divided by \( (x - a) \)’.
- Let \( f(x) \) be a polynomial of degree \( n \geq 1 \), then \( (x - a) \) is a factor of \( f(x) \) provided \( f(a) = 0 \). Also if \( (x + a) \) is a factor of \( f(x) \), then \( f(-a) = 0 \).
- \( x^3 + y^3 = (x + y)(x^2 - xy + y^2) \)
- \( x^3 - y^3 = (x - y)(x^2 + xy + y^2) \)
- \( x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx) \)
- If \( x + y + z = 0 \), then \( x^3 + y^3 + z^3 = 3xyz \)
- \( (x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx \)
- \( (x + y)^3 = x^3 + y^3 + 3xy(x + y) \)
- \( (x - y)^3 = x^3 - y^3 - 3xy(x - y) \)

VARIABLE
A symbol which can be assigned different numerical values is known as a variable. Variables are generally denoted by \( x, y, z, p, q, r, s \), etc.

CONSTANT
A symbol having a fixed value is called a constant, e.g. \( 8, 5, 9, \pi, a, b, c \), etc. are constants.

Note: The values of constants remain the same throughout a particular situation, but the value of a variable can keep changing.

ALGEBRAIC EXPRESSION
A combination of constants and variables, connected by some or all the basic operations \( +, -, \times, + \), is called an algebraic expression.

Example: \( 7 + 8x - 6x^2y - \frac{4}{9}xy \) is an algebraic expression.

TERMS
Various parts of an algebraic expression separated by \( (+) \) or \( (-) \) operations are called terms.

Examples: (i) In the above algebraic expression, terms are: 7, 8x, \(-6x^2y\) and \(-\frac{4}{9}xy\).
(ii) Various terms of \( 3p^4 - 6q^2 + 8r^3s - 2pq + 6s^3 \) are: \( 3p^4 \), \(-6q^2\), \( 8r^3s \), \(-2pq \) and \( 6s^3 \)
POLYNOMIAL
An algebraic expression in which the variables involved have non-negative integral powers is called a polynomial.

Examples: (i) \(x^3 + x^2 - 4x - 7\); (ii) \(3p^3 + 5p - 9\); (iii) \(x^2 + 2x\); etc. are all polynomials

Note: I. Each variable in a polynomial has a whole number as its exponent.
II. Each term of a polynomial has a co-efficient.

DEGREE OF A POLYNOMIAL
I. In case of a polynomial involving in one variable, the highest power of the variable is called the degree of the polynomial.
   Example: The degree of \(x^5 - 2x^3 + x\) is 5.

II. In case of a polynomial involving in more than one variable, the highest sum of exponents of variables in any term is called the degree of the polynomial.
   Example: The degree of \(p^2 - 6p^5q + 5p^2q^3 - 3q^4\) is 7
   \(\therefore\) The term \(-p^5q\) has the sum of exponents of \(p\) and \(q\) as 6 + 1, i.e. 7

TYPES OF POLYNOMIALS
A [On the basis of number of its terms]
   (i) Monomial: A polynomial containing only one non-zero term is called a monomial.
   (ii) Binomial: A polynomial containing two non-zero terms is called a binomial.
   (iii) Trinomial: A polynomial containing three non-zero terms is called a trinomial.

Note: I. A monomial containing a constant only is called a constant polynomial.
   II. A monomial containing its term as zero only is called a zero polynomial.
   III. If we add polynomials, we get a polynomial.
   IV. If we multiply polynomials, we get a polynomial.

B [On the basis of its degree]
   (i) Linear Polynomial: A polynomial of degree 1 is called a linear polynomial.
   (ii) Quadratic Polynomial: A polynomial of degree 2 is called a quadratic polynomial.
   (iii) Cubic Polynomial: A polynomial of degree 3 is called a cubic polynomial.
   (iv) Biquadratic Polynomial: A polynomial of degree 4 is called a biquadratic polynomial.

NCERT TEXTBOOK QUESTIONS SOLVED

EXERCISE 2.1 (Page 32)

Question 1. Which of the following expressions are polynomials in one variable and which are not?
State reasons for your answer:

\(4x^2 - 3x + 7\) \(y^2 + \sqrt{2}\) \(3\sqrt{7} + t\sqrt{2}\)

\(y + \frac{2}{y}\) \(x^{10} + y^3 + t^{50}\)

Solution: \(4x^2 - 3x + 7\)
\(\Rightarrow 4x^2 - 3x + 7x^0\)
(ii) \( y^2 + \sqrt{2} \)
\[ \Rightarrow y^2 + \sqrt{2} y^0 \]
\[ \therefore \text{All the exponents of } y \text{ are whole numbers.} \]
\[ \therefore y^2 + \sqrt{2} \text{ is a polynomial in one variable.} \]

(iii) \( 3t^{1/2} + t\sqrt{2} \)
\[ \Rightarrow 3t^{1/2} + \sqrt{2} \cdot t \]
\[ \therefore \frac{1}{2} \text{ is not a whole number,} \]
\[ \therefore 3t^{1/2} + \sqrt{2} \cdot t, \text{ i.e. } 3\sqrt{t} + t \cdot \sqrt{2} \text{ is not a polynomial.} \]

(iv) \( y + \frac{2}{y} \)
\[ \Rightarrow y + 2 \cdot y^{-1} \]
\[ \therefore (-1) \text{ is not a whole number.} \]
\[ \therefore y + 2y^{-1}, \text{ i.e. } y + \frac{2}{y} \text{ is not a polynomial.} \]

(v) \( x^{10} + y^{3} + t^{50} \)
\[ \therefore \text{Exponent of every variable is a whole number,} \]
\[ \therefore x^{10} + y^{3} + t^{50} \text{ is a polynomial in } x, \ y \text{ and } t, \text{ i.e. in three variables.} \]

**Question 2.** Write the co-efficients of \( x^2 \) in each of the following:

(i) \( 2 + x^2 + x \)
(ii) \( 2 - x^2 + x^3 \)
(iii) \( \frac{\pi}{2} x^2 + x \)
(iv) \( \sqrt{2}x - 1 \)

**Solution:**
(i) The co-efficient of \( x^2 \) is 1.
(ii) The co-efficient of \( x^2 \) is \(-1\).
(iii) The co-efficient of \( x^2 \) is \(\frac{\pi}{2}\).
(iv) \( \sqrt{2}x - 1 \Rightarrow \sqrt{2}x - 1 + 0 \cdot x^2 \)
\[ \therefore \text{The co-efficient of } x^2 \text{ is } 0. \]

**Question 3.** Give one example each of a binomial of degree 35, and of a monomial of degree 100.

**Solution:**
(i) A binomial of degree 35 can be: \( 3x^{35} - 4 \)
(ii) A monomial of degree 100 can be: \( \sqrt{2}y^{100} \)

**Question 4.** Write the degree of each of the following polynomials:

(i) \( 5x^3 + 4x^2 + 7x \)
(ii) \( 4 - y^2 \)
(iii) \( 5t - \sqrt{7} \)
(iv) \( 3 \)
Solution: 

(i) $5x^3 + 4x^2 + 7x$
\[ \therefore \text{The highest exponent of } x \text{ is 3.} \]
\[ \therefore \text{The degree of the polynomial is 3.} \]

(ii) $4 - y^2$
\[ \therefore \text{The highest exponent of } y \text{ is 2.} \]
\[ \therefore \text{The degree of the polynomial is 2.} \]

(iii) $5t - \sqrt{7}$
\[ \therefore \text{The highest exponent of } t \text{ is 1.} \]
\[ \therefore \text{The degree of the polynomial is 1.} \]

(iv) $3$
\[ \text{since, } 3 = 3x^0 \]
\[ \therefore \text{The degree of the polynomial 3 is 0.} \]

Question 5. Classify the following as linear, quadratic and cubic polynomials:

(i) $x^2 + x$
(ii) $x - x^3$
(iii) $y + y^2 + 4$
(iv) $1 + x$
(v) $3t$
(vi) $r^2$
(vii) $7x^3$

Solution: 

(i) $x^2 + x$
\[ \therefore \text{The degree of } x^2 + x \text{ is 2.} \]
\[ \therefore \text{It is a quadratic polynomial.} \]

(ii) $x - x^3$
\[ \therefore \text{The degree of } x - x^3 \text{ is 3.} \]
\[ \therefore \text{It is a cubic polynomial.} \]

(iii) $y + y^2 + 4$
\[ \therefore \text{The degree of } y + y^2 + 4 \text{ is 2.} \]
\[ \therefore \text{It is a quadratic polynomial.} \]

(iv) $1 + x$
\[ \therefore \text{The degree of } 1 + x \text{ is 1.} \]
\[ \therefore \text{It is a linear polynomial.} \]

(v) $3t$
\[ \therefore \text{The degree of } 3t \text{ is 1.} \]
\[ \therefore \text{It is a linear polynomial.} \]

(vi) $r^2$
\[ \therefore \text{The degree of } r^2 \text{ is 2.} \]
\[ \therefore \text{It is a quadratic polynomial.} \]

(vii) $7x^3$
\[ \therefore \text{The degree of } 7x^3 \text{ is 3.} \]
\[ \therefore \text{It is a cubic polynomial.} \]
TEST YOUR SKILLS

1. Fill in the blanks:
   (i) The degree of a non-zero constant polynomial is __________.
   (ii) A polynomial having three terms is called __________.
   (iii) A polynomial of degree two is called a __________ polynomial.
   (iv) A polynomial of degree __________ is called a linear polynomial.

2. Find the degree of each of the following polynomials:
   (i) \( t^5 - t^4 + 5 \)
   (ii) \( 2x^8 + y^3 - y^2 + 2 \)
   (iii) \( 2x \)
   (iv) \( 2 \)

3. Classify the following polynomials as monomial, binomial or trinomial:
   (i) \( 5x^2 - 3x^2 + 6 \)
   (ii) \( 7 \)
   (iii) \( 1 - x^3 \)
   (iv) \( 38 \)
   (v) \( 7x^2 \)

4. Write the co-efficient of ‘x’ in the following polynomials:
   (i) \( 4x^2 - \frac{3}{2}x - 7 \)
   (ii) \( \sqrt{5}x \)
   (iii) \( 2x \)
   (iv) \( 18x^{15} - x^{10} + 2x \)

ANSWERS

1. (i) zero (ii) trinomial (iii) quadratic (iv) one
2. (i) 5 (ii) 8 (iii) 1 (iv) 0
3. (i) trinomial (ii) monomial (iii) binomial (iv) monomial
   (v) monomial
4. (i) \( -\frac{3}{2} \) (ii) \( \sqrt{5} \) (iii) 2 (iv) 2

ZEROS OF A POLYNOMIAL

Let \( p(x) \) is a polynomial. If \( p(a) = 0 \), then ‘a’ is said to be a zero of the polynomial \( p(x) \).

Note: (i) Finding the ‘zeros’ of a polynomial means solving the equation \( p(x) = 0 \).
(ii) A non-zero ‘constant polynomial’ has no zero.
(iii) Every real number is a zero of the zero polynomial.
(iv) Every linear polynomial has one and only one zero.
(v) A polynomial can have more than one zero.
(vi) A zero of a polynomial need not be 0.
(vii) ‘0’ may be a zero of a polynomial.

NCERT TEXTBOOK QUESTIONS SOLVED

EXERCISE 2.2 (Page 34)

Question 1. Find the value of the polynomial \( 5x - 4x^2 + 3 \) at
   (i) \( x = 0 \)  
   (ii) \( x = -1 \)  
   (iii) \( x = 2 \)

Solution: (i) ∴ \( p(x) = 5x - 4x^2 + 3 = 5(x) - 4(x)^2 + 3 \)
   ∴ \( p(0) = 5(0) - 4(0)^2 + 3 = 0 - 0 + 3 = 3 \)
   Thus, the value of \( 5x - 4x^2 + 3 \) at \( x = 0 \) is 3.

   (ii) ∴ \( p(x) = 5x - 4x^2 + 3 = 5(x) - 4(x)^2 + 3 \)
The value of $5x - 4x^2 + 3$ at $x = -1$ is $-6$.

$$p(x) = 5x - 4x^2 + 3 = 5(x) - 4(x)^2 + 3$$

Thus the value of $5x - 4x^2 + 3$ at $x = 2$ is $-3$.

**Question 2.** Find $p(0)$, $p(1)$ and $p(2)$ for each of the following polynomials:

(i) $p(y) = y^2 - y + 1$

(ii) $p(t) = 2 + t + 2t^2 - t^3$

(iii) $p(x) = x^3$

(iv) $p(x) = (x - 1)(x + 1)$

**Solution:**

(i) $p(y) = y^2 - y + 1$

$$p(0) = (0)^2 - (0) + 1 = 0 - 0 + 1 = 1$$

$$p(1) = (1)^2 - (1) + 1 = 1 - 1 + 1 = 1$$

$$p(2) = (2)^2 - 2 + 1 = 4 - 2 + 1 = 3$$

(ii) $p(t) = 2 + t + 2t^2 - t^3$

$$p(0) = 2 + (0) + 2(0)^2 - (0)^3 = 2 + 0 + 0 - 0 = 2$$

$$p(1) = 2 + (1) + 2(1)^2 - (1)^3 = 2 + 1 + 2 - 1 = 4$$

$$p(2) = 2 + 2 + 2(2)^2 - (2)^3 = 2 + 2 + 8 - 8 = 4$$

(iii) $p(x) = x^3$

$$p(0) = (0)^3 = 0$$

$$p(1) = (1)^3 = 1$$

$$p(2) = (2)^3 = 8 \quad [\therefore 2 \times 2 \times 2 = 8]$$

(iv) $p(x) = (x - 1)(x + 1)$

$$p(0) = (0 - 1)(0 + 1) = -1 \times 1 = -1$$

$$p(1) = (1 - 1)(1 + 1) = (0)(2) = 0$$

$$p(2) = (2 - 1)(2 + 1) = (1)(3) = 3$$

**Question 3.** Verify whether the following are zeros of the polynomial, indicated against them.

(i) $p(x) = 3x + 1$, $x = -\frac{1}{3}$

(ii) $p(x) = 5x - \pi$, $x = \frac{4}{5}$

(iii) $p(x) = x^2 - 1$, $x = 1, -1$

(iv) $p(x) = (x + 1)(x - 2)$, $x = -1, 2$

(v) $p(x) = x^2$, $x = 0$

(vi) $p(x) = lx + m$, $x = \frac{-m}{l}$

(vii) $p(x) = 3x^2 - 1$, $x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

(viii) $p(x) = 2x + 1$, $x = \frac{1}{2}$
Solution: (i) \( \therefore \) \( p(x) = 3x + 1 \)

\[ p\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right) + 1 = -1 + 1 = 0 \]

\[ \therefore x = -\frac{1}{3} \] is the zero of \( 3x + 1 \).

(ii) Since,

\[ p(x) = 5x - \pi \]

\[ p\left(\frac{4}{5}\right) = 5\left(\frac{4}{5}\right) - \pi = 4 - \pi \]

\[ \therefore p\left(\frac{4}{5}\right) \neq 0 \]

\[ \therefore x = \frac{4}{5} \] is not the zero of \( 5x - \pi \).

(iii) Since,

\[ p(x) = x^2 - 1 \]

\[ p(1) = (1)^2 - 1 = 1 - 1 = 0 \]

Since,

\[ p(1) = 0, \]

\[ \therefore x = 1 \] is a zero of \( x^2 - 1 \).

Also \[ p(-1) = (-1)^2 - 1 = 1 - 1 = 0 \]

i.e., \[ p(-1) = 0, \]

\[ \therefore x = -1 \] is also a zero of \( x^2 - 1 \).

(iv) We have

\[ p(x) = (x + 1)(x - 2) \]

\[ \therefore p(-1) = (-1 + 1)(-1 - 2) = (0)(-3) = 0 \]

Since \[ p(-1) = 0, \]

\[ \therefore x = -1 \] is a zero of \((x + 1)(x - 1)\).

Also, \[ p(2) = (2 + 1)(2 - 2) = (3)(0) = 0 \]

Since \[ p(2) = 0, \]

\[ \therefore x = 2 \] is also a zero of \((x + 1)(x - 1)\).

(v) We have

\[ p(x) = x^2 \]

\[ \therefore p(0) = (0)^2 = 0 \]

Since \[ p(0) = 0, \]

\[ \therefore 0 \] is a zero of \( x^2 \).

(vi) We have

\[ p(x) = lx + m \]

\[ \therefore p\left(-\frac{m}{l}\right) = l\left(-\frac{m}{l}\right) + m = -m + m = 0 \]

Since \[ p\left(-\frac{m}{l}\right) = 0, \]

\[ \therefore x = \left(-\frac{m}{l}\right) \] is a zero of \( lx + m \).

(vii) We have

\[ p(x) = 3x^2 - 1 \]

\[ \therefore p\left(-\frac{1}{\sqrt{3}}\right) = 3\left(-\frac{1}{\sqrt{3}}\right)^2 - 1 = 3\left(\frac{1}{3}\right) - 1 \]

\[ = 1 - 1 = 0 \]
Since \( p\left(\frac{1}{\sqrt{3}}\right) = 0, \)

\( \therefore \frac{1}{\sqrt{3}} \) is a zero of \( x^2 - 1. \)

Also, \( p\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 1 \)

\[ = 3\left(\frac{4}{3}\right) - 1 = 4 - 1 = 3 \]

Since, \( p\left(\frac{2}{\sqrt{3}}\right) \neq 0, \)

\( \therefore \frac{2}{\sqrt{3}} \) is not a zero of \( x^2 - 1. \)

(viii) We have \( p(x) = 2x + 1 \)

\( \therefore p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + 1 + 1 = 2 \)

Since \( p\left(\frac{1}{2}\right) \neq 0, \)

\( \therefore x = \frac{1}{2} \) is not a zero of \( 2x + 1. \)

**Question 4.** Find the zero of the polynomial in each of the following cases:

- (i) \( p(x) = x + 5 \)
- (ii) \( p(x) = x - 5 \)
- (iii) \( p(x) = 2x + 5 \)
- (iv) \( p(x) = 3x - 2 \)
- (v) \( p(x) = 3x \)
- (vi) \( p(x) = ax, a \neq 0 \)
- (vii) \( p(x) = cx + d, c \neq 0, c, d \) are real numbers.

**Solution:**

(i) We have \( p(x) = x + 5 \)

\( \therefore p(x) = 0 \)

\( \Rightarrow x + 5 = 0 \)

or \( x = -5 \)

Thus, a zero of \( x + 5 \) is \(-5\).

(ii) We have \( p(x) = x - 5 \)

\( \therefore p(x) = 0 \)

\( \Rightarrow x - 5 = 0 \)

or \( x = 5 \)

Thus, a zero of \( x - 5 \) is \(5\).

(iii) We have \( p(x) = 2x + 5 \)

\( \therefore p(x) = 0 \)

\( \Rightarrow 2x + 5 = 0 \)

or \( 2x = -5 \)

or \( x = \frac{-5}{2} \)

Thus, a zero of \( 2x + 5 \) is \(\frac{-5}{2}\).
(iv) Since \( p(x) = 3x - 2 \)
\[ \therefore \quad p(x) = 0 \]
\[ \Rightarrow \quad 3x - 2 = 0 \]
or \[ 3x = 2 \]
or \[ x = \frac{2}{3} \]
Thus, a zero of \( 3x - 2 \) is \( \frac{2}{3} \).

(v) Since \( p(x) = 3x \)
\[ \therefore \quad p(x) = 0 \]
\[ \Rightarrow \quad 3x = 0 \]
or \[ x = \frac{0}{3} \text{ or } 0 \]
Thus, a zero of \( 3x \) is \( 0 \).

(vi) Since, \( p(x) = ax, a \neq 0 \)
\[ \therefore \quad p(x) = 0 \]
\[ \Rightarrow \quad ax = 0 \]
or \[ x = \frac{0}{a} = 0 \]
Thus, a zero of \( ax \) is \( 0 \).

(vii) Since, \( p(x) = cx + d \)
\[ \therefore \quad p(x) = 0 \]
\[ \Rightarrow \quad cx + d = 0 \]
or \[ cx = -d \]
or \[ x = \frac{-d}{c} \]
Thus, a zero of \( cx + d \) is \( \frac{-d}{c} \).

**Question 5.** If \( p(x) = x^2 - 4x + 3 \), evaluate: \( p(2) - p(-1) + p\left(\frac{-1}{2}\right) \)

**Solution:**

We have \( p(x) = x^2 - 4x + 3 \)
\[ \therefore \quad p(-1) = (-1)^2 - 4(-1) + 3 \]
\[ = 1 + 4 + 3 = 8 \]
and \( p(2) = (2)^2 - 4(2) + 3 \)
\[ = 4 - 8 + 3 = -1 \]
also \( p\left(\frac{-1}{2}\right) = \left(\frac{-1}{2}\right)^2 - 4 \left(\frac{-1}{2}\right) + 3 \)
\[ = \frac{1}{4} + 2 + 3 \]
\[ = 5 + \frac{1}{4} = \frac{21}{4} \]
Thus, \( p(2) - p(-1) + p\left(\frac{-1}{2}\right) \)
\[ = -1 - 8 + \frac{21}{4} = -9 + \frac{21}{4} = \frac{-36 + 21}{4} \]
\[ = \frac{-15}{4} \]

**TEST YOUR SKILLS**

1. Find the value of \( 5x^3 - 2x^2 + 3x - 2 \) at
   (i) \( x = 1 \)
   (ii) \( x = 0 \)
   (iii) \( x = -1 \)

2. Find the value of each of the following polynomials at the indicated value of variables:
   (i) \( p(x) = 7 - 3x + 5x^2 \), at \( x = 1 \)
   (ii) \( q(y) = \sqrt{11} - 4y + 3y^3 \), at \( y = 2 \)
   (iii) \( p(t) = 6 - t + 4t^2 \), at \( t = a \)

3. (i) Is \( -2 \) a zero of \( x + 2 \)?
   (ii) Is \( 2 \) a zero of \( x + 2 \)?

4. Find a zero of \( p(x) = 2x + 1 \)

5. Prove that \( 2 \) and \( 0 \) are zeros of \( p(x) = x^2 - 2x \).

6. If \( p(x) = x + 3 \), Then \( p(x) + p(-x) \) is equal to:
   (i) \( 3 \)
   (ii) \( 2x \)
   (iii) \( 0 \)
   (iv) \( 6 \)

7. If \( p(x) = x^2 - 2\sqrt{2}x + 1 \), Then \( p\left(2\sqrt{2}\right) \) is equal to:
   (i) \( 0 \)
   (ii) \( 1 \)
   (iii) \( 4\sqrt{2} \)
   (iv) \( 8\sqrt{2} + 1 \)

8. The value of the polynomial \( 5x - 4x^2 + 3 \), when \( x = -1 \) is:
   (i) \( -6 \)
   (ii) \( 6 \)
   (iii) \( 2 \)
   (iv) \( -2 \)

**ANSWERS**

1. (i) 4  \hspace{1cm} (ii) 2  \hspace{1cm} (iii) -12
2. (i) 9  \hspace{1cm} (ii) \( 16 - \sqrt{11} \)  \hspace{1cm} (iii) \( 4a^2 - a + 6 \)
3. (i) Yes  \hspace{1cm} (ii) No
4. \( \left(\frac{1}{2}\right) \)
6. 6  \hspace{1cm} 7  \hspace{1cm} 1  \hspace{1cm} 8  \hspace{1cm} -6

**REMAINDER THEOREM**

We know that:

\[
\text{Dividend} = (\text{Divisor} \times \text{Quotient}) + \text{Remainder}
\]

If in general, we denote:

- Dividend as \( p(x) \),
- Divisor as \( g(x) \),
- Quotient as \( q(x) \),
- Remainder as \( r(x) \).
Such that: (i) $p(x)$, $g(x)$, $q(x)$ and $r(x)$ are polynomials, and
(ii) $g(x) \neq 0$
(iii) Degree of $r(x) < \text{Degree of } g(x)$
(iv) $\text{Degree of } g(x) \leq \text{Degree of } p(x)$

Then, we have
$$p(x) = [g(x) \cdot q(x)] + r(x)$$

**Note:** Theorem means a statement which is already proved.

**Theorem:**
Let $p(x)$ be any polynomial of degree greater than or equal to one and let ‘$a$’ be any real number. If $p(x)$ is divided by the linear polynomial $(x – a)$, then the remainder is $p(a)$.

**Proof:**
Let $p(x)$ be divided by $(x – a)$ such that:

- Quotient = $q(x)$,
- Remainder = $r(x)$

∴
$$p(x) = [(x – a) \cdot q(x)] + r(x) \quad \ldots(1)$$

Since, remainder is always smaller than the divisor.
∴ Divisor $(x – a)$ is having degree as 1.
∴ Degree of remainder must be zero.

We know that the degree of a constant polynomial is zero.
∴ $r(x)$ is a constant, say ‘$r$’.

Now, from (1), we have

$$p(x) = [(x – a) \cdot q(x)] + r$$

If
$$x = a \text{ then } P(a) = [(a – a) \cdot q(a)] + r$$

or
$$p(a) = [0 \times q(a)] + r$$

or
$$p(a) = 0 + r$$

or
$$p(a) = r$$

i.e.
$$\text{Remainder} = p(a).$$

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**NCERT TEXTBOOK QUESTIONS SOLVED**

**EXERCISE 2.3 (Page 40)**

**Question 1.** Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by

(i) $x + 1$  
(ii) $x - \frac{1}{2}$  
(iii) $x$

(iv) $x + \pi$  
(v) $5 + 2x$

**Solution:** (i) ∴ The zero of $x + 1$ is $-1$

\[∴ x + 1 = 0 \Rightarrow x = -1 \]

And by remainder theorem, when $p(x) = x^3 + 3x^2 + 3x + 1$ is divided by $x + 1$, then remainder is $p(-1)$. 

---

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\[ p(-1) = (-1)^3 + 3(-1)^2 + 3(-1) + 1 \\
= -1 + 3 + (-3) + 1 \\
= 0 \]

Thus, the required remainder = 0

(ii) \[ : \text{The zero of } x - \frac{1}{2} \text{ is } \frac{1}{2} \]

and \[ p(x) = x^3 + 3x^2 + 3x + 1 \]

\[ \therefore \text{ For divisor } = x - \frac{1}{2}, \text{ remainder is given as} \]

\[ p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) + 1 = \frac{1}{8} + \frac{3}{4} + \frac{3}{2} + 1 \]

\[ = \frac{1}{8} + \frac{12}{8} + \frac{8}{8} = \frac{27}{8} \]

Thus, the required remainder = \(\frac{27}{8}\).

(iii) We have \[ p(x) = x^3 + 3x^2 + 3x + 1 \]

and the zero of \(x \) is \(0\)

\[ \therefore \]

\[ p(0) = (0)^3 + 3(0)^2 + 3(0) + 1 = 0 + 0 + 0 + 1 = 1 \]

Thus, the required remainder = 1.

(iv) We have \[ p(x) = x^3 + 3x^2 + 3x + 1 \]

and zero of \(x + \pi = (-\pi)\)

\[ \therefore \]

\[ p(-\pi) = (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1 \]

\[ = -\pi^3 + 3\pi^2 - 3\pi + 1 \]

Thus, the required remainder is \(-\pi^3 + 3\pi^2 - 3\pi + 1\).

(v) We have \[ p(x) = x^3 + 3x^2 + 3x + 1 \]

and zero of \(5 + 2x = \left(-\frac{5}{2}\right)\)

\[ \therefore \]

\[ p\left(-\frac{5}{2}\right) = \left[-\frac{5}{2}\right]^3 + 3\left[-\frac{5}{2}\right]^2 + 3\left[-\frac{5}{2}\right] + 1 \]

\[ = -\frac{125}{8} + \frac{75}{2} - \frac{15}{2} + 1 \]

\[ = -\frac{125}{8} + \frac{150}{8} - \frac{120}{8} + 1 \]

Thus, the required remainder is \(-\frac{27}{8}\).

**Question 2.** Find the remainder when \(x^3 - ax^2 + 6x - a\) is divided by \(x - a\).

**Solution:** We have \[ p(x) = x^3 - ax^2 + 6x - a \]

\[ \therefore \text{ Zero of } x - a \text{ is } a. \]

\[ \therefore x - a = 0 \Rightarrow x = a \]
\[ p(a) = (a)^3 - a(a)^2 + 6(a) - a = a^3 - a^3 + 6a - a \]
\[ = 0 + 5a = 5a \]
Thus, the required remainder = 5a

**Question 3.** Check whether 7 + 3x is a factor of 3x^3 + 7x.

**Solution:** We have \( p(x) = 3x^3 + 7x \) and zero of 7 + 3x is \( \frac{-7}{3} \)
\[ \because 7 + 3x = 0 \Rightarrow x = \frac{-7}{3} \]
\[ \therefore p\left(\frac{-7}{3}\right) = 3\left(\frac{-7}{3}\right)^3 + 7\left(\frac{-7}{3}\right) = 3\left(\frac{-343}{27}\right) + \left(\frac{-49}{3}\right) \]
\[ = \frac{-343}{9} - \frac{49}{3} = \frac{-490}{9} \]
Since \( \frac{-490}{9} \neq 0 \)
i.e. the remainder is not 0.
\[ \therefore 3x^3 - 7x \text{ is not divisible by } 7 + 3x. \]
Thus, (7 + 3x) is not a factor of 3x^3 - 7x.

**TEST YOUR SKILLS**

1. Find the remainder when:
   (i) \( 3x^2 + x - 1 \) is divided by \( x + 1 \)
   (ii) \( 3x^4 - 3x^3 - 3x - 1 \) is divided by \( x - 1 \)
   (iii) \( x^3 + 1 \) divided by \( x + 1 \)
   (iv) \( x^4 + x^3 - 2x^2 + x + 1 \) is divided by \( x - 1 \)

2. Check whether the polynomial \( q(t) = 4t^3 + 4t^2 - t - 1 \) is a multiple of \( 2t + 1 \).
   
   **Hint:** If \( 2t + 1 \) is a factor of \( q(t) \), then \( q(t) \) is a multiple of \( 2t + 1 \).

**ANSWERS**

<table>
<thead>
<tr>
<th></th>
<th>(i)</th>
<th>(ii)</th>
<th>(iii)</th>
<th>(iv)</th>
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<tbody>
<tr>
<td>1.</td>
<td>1</td>
<td>-4</td>
<td>0</td>
<td>2</td>
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</table>

**FACTOR THEOREM**

If \( p(x) \) is any polynomial of degree 1 or greater than 1 and ‘a’ be any real number such that:
(i) \( (x - a) \) is a factor of \( p(x) \), then \( p(a) = 0 \); and
(ii) \( p(a) = 0 \), then \( (x - a) \) is a factor of \( p(x) \).

**FACTORISATION OF POLYNOMIALS**

[Using the “splitting the middle-term” method]
Let \( ax^2 + bx + c \) be a quadratic polynomial such that \( a \), \( b \) and \( c \) are constants and \( a \neq 0 \).
To factorise it, we split the co-efficient of \( x \) into two parts ‘l’ and ‘m’ such that
\[ l + m = b \]
\[ l \times m = a \times c \]
**NCERT TEXTBOOK QUESTIONS SOLVED**

**EXERCISE 2.4 (Page 43)**

**Question 1.** Determine which of the following polynomials has a factor \((x + 1)\):

(i) \(x^3 + x^2 + x + 1\)

(ii) \(x^4 + x^3 + x^2 + x + 1\)

(iii) \(x^4 + 3x^3 + 3x^2 + x + 1\)

(iv) \(x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}\)

**Solution:** For \(x + 1 = 0\), we have \(x = -1\).

\(\therefore\) The zero of \(x + 1\) is \(-1\).

(i) \[
p(x) = x^3 + x^2 + x + 1
p(-1) = (-1)^3 + (-1)^2 + (-1) + 1
= -1 + 1 - 1 + 1 = 0
\]

\(\therefore\) \((x + 1)\) is a factor of \(x^3 + x^2 + x + 1\).

(ii) \[
p(x) = x^4 + x^3 + x^2 + x + 1
p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1
= 1 - 1 + 1 - 1 + 1
= 1
\]

\(\therefore \) \(p(1) \neq 0\)

\(\therefore\) \((x + 1)\) is not a factor of \(x^4 + x^3 + x^2 + x + 1\).

(iii) \[
p(x) = x^4 + 3x^3 + 3x^2 + x + 1
p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1
= 1 - 3 + 3 - 1 + 1
= 1 \neq 0
\]

\(\therefore \) \(p(1) \neq 0\)

\(\therefore\) \((x + 1)\) is not a factor of \(x^4 + 3x^3 + 3x^2 + x + 1\).

(iv) \[
p(x) = x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}
\]

\[
p(-1) = (-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) + \sqrt{2}
= -1 - 1 - (-1)(2 + \sqrt{2}) + \sqrt{2}
= -2 + 2 + \sqrt{2} + \sqrt{2}
= 2\sqrt{2} \neq 0
\]

Since \(p(-1) \neq 0\).

\(\therefore\) \((x + 1)\) is not a factor of \(x^4 + 3x^3 + 3x^2 + x + 1\).
**Question 2.** Use the factor theorem to determine whether \( g(x) \) is a factor of \( p(x) \) in each of the following cases:

(i) \( p(x) = 2x^3 + x^2 - 2x - 1, \ g(x) = x + 1 \)

(ii) \( p(x) = x^3 + 3x^2 + 3x + 1, \ g(x) = x + 2 \)

(iii) \( p(x) = x^3 - 4x^2 + x + 6, \ g(x) = x - 3 \)

**Solution:**

(i) We have \( p(x) = 2x^3 + x^2 - 2x - 1 \) and \( g(x) = x + 1 \)

\[
p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1 = -2 + 1 + 2 - 1 = 0
\]

\( \therefore \) \( g(x) \) is a factor of \( p(x) \).

(ii) We have \( p(x) = x^3 + 3x^2 + 3x + 1 \) and \( g(x) = x + 2 \)

\[
p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1 = -8 + 12 - 6 + 1 = -1 \]

\( \therefore \) \( p(-2) \neq 0 \)

Thus, \( g(x) \) is not a factor of \( p(x) \).

(iii) We have \( p(x) = x^3 - 4x^2 + x + 6 \) and \( g(x) = x - 3 \)

\[
p(3) = (3)^3 - 4(3)^2 + (3) + 6 = 27 - 36 + 3 + 6 = 0
\]

Since \( g(x) = 0 \)

\( \therefore \) \( g(x) \) is a factor of \( p(x) \).

**Question 3.** Find the value of \( k \), if \( x - 1 \) is a factor of \( p(x) \) in each of the following cases:

(i) \( p(x) = x^2 + x + k \)  
(ii) \( p(x) = 2x^2 + kx + \sqrt{2} \)  
(iii) \( p(x) = kx^2 - \sqrt{2}x + 1 \)  
(iv) \( p(x) = kx^2 - 3x + k \)

**Solution:**

(i) Here \( p(x) = x^2 + x + k \)

For \( x - 1 \) to be a factor of \( p(x) \), \( p(1) \) should be equal to 0.

We have \( p(1) = (1)^2 + 1 + k \)

or \( k + 2 = 0 \)

\( \Rightarrow \) \( k = -2 \)

(ii) Here, \( p(x) = 2x^2 + kx + \sqrt{2} \)

For \( x - 1 \) to be a factor of \( p(x) \), \( p(1) = 0 \)

Since, \( p(1) = 2(1)^2 + k(1) + \sqrt{2} = 2 + k + \sqrt{2} \)

\( \therefore \) \( p(1) \) must be equal to 0.

\( k + 2 + \sqrt{2} = 0 \)

\( \Rightarrow \) \( k = -2 - \sqrt{2} \)

or \( k = -(2 + \sqrt{2}) \).
(iii) Here \( p(x) = kx^2 - \sqrt{2}x + 1 \) and \( g(x) = x - 1 \)
\[ \therefore \text{For } (x - 1) \text{ be a factor of } p(x), \text{ } p(1) \text{ should be equal to } 0. \]
Since \[ p(1) = k(1)^2 - \sqrt{2}(1) + 1 \]
or \[ p(1) = k - \sqrt{2} + 1 \]
or \[ p(1) = k - \sqrt{2} + 1 \]
\[ \therefore k - \sqrt{2} + 1 = 0 \]
\[ \Rightarrow \quad k = \sqrt{2} - 1 \]

(iv) Here \( p(x) = kx^2 - 3x + k \) and \( g(x) = x - 1 \)
For \( g(x) \) be a factor of \( p(x) \), \( p(1) \) should be equal to 0.
Since \[ p(1) = k(1)^2 - 3(1) + k \]
\[ = k - 3 + k \]
\[ = 2k - 3 \]
\[ \therefore \quad 2k - 3 = 0 \]
\[ \Rightarrow \quad k = \frac{3}{2} \]

**Question 4. Factorise:**

(i) \( 12x^2 - 7x + 1 \)
(ii) \( 2x^2 + 7x + 3 \)
(iii) \( 6x^2 + 5x - 6 \)
(iv) \( 3x^2 - x - 4 \)

**Solution:**

(i) \( 12x^2 - 7x + 1 \)
Here co-efficient of \( x^2 = 12 \)
Co-efficient of \( x = -7 \)
and constant term = 1
\[ \therefore a = 12, \ b = -7, \ c = 1 \]
Now, \[ l + m = -7 \text{ and } lm = ac = 12 \times 1 \]
\[ \therefore \text{We have } l = (-4) \text{ and } m = (-3) \]
i.e. \[ b = -7 = (-4 - 3), \]
Now, \[ 12x^2 - 7x + 1 = 12x^2 - 4x - 3x + 1 \]
\[ = 4x(3x - 1) - 1(3x - 1) \]
\[ = (3x - 1)(4x - 1) \]
Thus, \[ 12x^2 - 7x + 1 = (3x - 1)(4x - 1) \]

(ii) \( 2x^2 + 7x + 3 \)
Here, \[ a = 2, \ b = 7 \text{ and } c = 3 \]
\[ \therefore \text{We have } l + m = 7 \text{ and } lm = 2 \times 3 = 6 \]
i.e. \[ l + 6 = 7 \text{ and } 1 \times 6 = 6 \]
\[ \therefore \text{We have } l = 1 \text{ and } m = 6 \]
Thus, \[ 2x^2 + 7x + 3 = (2x + 1)(x + 3) \]

(iii) \( 6x^2 + 5x - 6 \)
We have \[ a = 6, \ b = 5 \text{ and } c = -6 \]
\[ \therefore \text{We have } l + m = 5 \text{ and } lm = ac = 6 \times (-6) = -36 \]
\. \( l + m = 9 + (-4) \)
\[ \therefore 6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6 \]
\[ = 3(2x + 3) - 2(2x + 3) \]
\[ = (2x + 3)(3x - 2) \]

Thus,
\[ 6x^2 + 5x - 6 = (2x + 3)(3x - 2) \]

(iv) \( 3x^2 - x - 4 \)

We have \( a = 3, \ b = -1 \) and \( c = -4 \)
\[ \therefore l + m = -1 \) and \( lm = 3 \times (-4) = -12 \]
\[ \therefore l = -4 \) and \( m = 3 \)

Now,
\[ 3x^2 - x - 4 = 3x^2 - 4x + 3x - 4 \]
\[ = x(3x - 4) + 1(3x - 4) \]
\[ = (3x - 4)(x + 1) \]

Thus,
\[ 3x^2 - x - 4 = (3x - 4)(x + 1) \]

**Question 5. Factorise:**

(i) \( x^3 - 2x^2 - x + 2 \)

(ii) \( x^3 - 3x^2 - 9x - 5 \)

(iii) \( x^3 + 13x^2 + 32x + 20 \)

(iv) \( 2y^3 + y^2 - 2y - 1 \)

**Solution:**

(i) \( x^3 - 2x^2 - x + 2 \)

Rearranging the terms, we have
\[ x^3 - 2x^2 - x + 2 = x^3 - x - 2x^2 + 2 \]
\[ = x(x^2 - 1) - 2(x^2 - 1) \]
\[ = (x^2 - 1)(x - 2) \]
\[ = [(x)^2 - (1)^2][x - 2] \]
\[ = (x - 1)(x + 1)(x - 2) \]

\[ \therefore a^2 - b^2 = (a + b)(a - b) \]

Thus,
\[ x^3 - 2x^2 - x + 2 = (x - 1)(x + 1)(x - 2) \]

(ii) \( x^3 - 3x^2 - 9x - 5 \)

We have \( p(x) = x^3 - 3x^2 - 9x - 5 \)

By trial, let us find:
\[ p(1) = (1)^3 - 3(1)^2 - 9(1) - 5 \]
\[ = 1 - 3 - 9 - 5 = -14 \neq 0 \]

Now
\[ p(-1) = (-1)^3 - 3(-1)^2 - 9(-1) - 5 \]
\[ = -1 - 3 + 9 - 5 = 0 \]

\[ \therefore \text{By factor theorem, } [x - (-1)] \text{ is a factor of } p(x). \]

Now,
\[ \frac{x^3 - 3x^2 - 9x - 5}{x - (-1)} = x^2 - 4x - 5 \]

\[ \therefore x^2 - 3x^2 - 9x - 5 = (x + 1)(x^2 - 4x - 5) \]
\[ = (x + 1)[x^2 - 5x + x - 5] \]

[Splitting -4 into -5 and +1]
\[ = (x + 1) [x(x - 5) + 1(x - 5)] \]
\[ = (x + 1) [(x - 5) (x + 1)] \]
\[ = (x + 1)(x - 5)(x + 1) \]

(iii) \( x^3 + 13x^2 + 32x + 20 \)

We have \( p(x) = x^3 + 13x^2 + 32x + 20 \)

By trial, let us find:
\[ p(1) = (1)^3 + 13(1)^2 + 32(1) + 20 \]
\[ = 1 + 13 + 32 + 20 = 66 \neq 0 \]
Now \[ p(-1) = (-1)^3 + 13(-1)^2 + 32(-1) + 20 \]
\[ = -1 + 13 - 32 + 20 \]
\[ = 0 \]

\[ \therefore \text{By factor theorem, } [x - (-1)], \text{ i.e. } (x + 1) \text{ is a factor } p(x). \]

\[ \therefore \frac{x^3 - 13x^2 - 32x - 20}{(x+1)} = x^2 + 12x + 20 \]
or \[ x^3 + 13x^2 + 32x + 20 = (x + 1)(x^2 + 12x + 20) \]
\[ = (x + 1)[x^2 + 2x + 10x + 20] \]
\[ = (x + 1)[x(x + 2) + 10(x + 2)] \]
\[ = (x + 1)(x + 2)(x + 10) \]

(iv) \[ 2y^3 + y^2 - 2y - 1 \]
We have \[ p(y) = 2y^3 + y^2 - 2y - 1 \]
By trial, we have \[ p(1) = 2(1)^3 + (1)^2 - 2(1) - 1 \]
\[ = 2(1) + 1 - 2 - 1 \]
\[ = 2 + 1 - 2 - 1 = 0 \]

\[ \therefore \text{By factor theorem, } (y - 1) \text{ is a factor of } p(y). \]

\[ \therefore \frac{(2y^3 + y^2 - 2y - 1)}{(y - 1)} = 2y^2 + 3y + 1 \]
\[ \therefore 2y^3 - y^2 - 2y - 1 = (y - 1)(2y^2 + 3y + 1) \]
\[ = (y - 1)[2y^2 + 2y + y + 1] \]
\[ = (y - 1)[2y(y + 1) + 1(y + 1)] \]
\[ = (y - 1)(y + 1)(2y + 1) \]

**ADDITIONAL QUESTIONS SOLVED**

**Question 1.** Show that \((x + 3)\) is a factor of \(x^3 + x^2 - 4x + 6\).

**Solution:** \[ p(x) = x^3 + x^2 - 4x + 6 \]
Since \((x + 3)\) is a factor, then \(x + 3 = 0 \Rightarrow x = -3 \)

\[ \therefore p(-3) = (-3)^3 + (-3)^2 - 4(-3) + 6 \]
\[ = -27 + 9 + 12 + 6 \]
\[ = 0 \]

\[ \therefore (x + 3) \text{ is a factor of } x^3 + x^2 - 4x + 6 \]

**Question 2.** Show that \((x - 5)\) is a factor of: \(x^3 - 3x^2 - 13x + 15\)

**Solution:** \[ p(x) = x^3 - 3x^2 - 13x + 15 \]
Since \((x - 5)\) is a factor, then \(x - 5 = 0 \Rightarrow x = 5 \)

\[ \therefore p(5) = (5)^3 - 3(5)^2 - 13(5) + 15 \]
\[ = 125 - 75 - 65 + 15 \]
\[ = 140 - 140 = 0 \]

\[ \therefore (x - 5) \text{ is a factor of } x^3 - 3x^2 - 13x + 15. \]
Question 3. Find the value of $\alpha$ such that $(x + \alpha)$ is a factor of the polynomial

\[ f(x) = x^4 - \alpha^2 x^2 + 2x + \alpha + 3. \]

Solution: Here $f(x) = x^4 - \alpha^2 x^2 + 2x + \alpha + 3$

Since, $(x + \alpha)$ is a factor of $f(x)$

\[ \therefore f(-\alpha) = 0 \]

\[ \Rightarrow (-\alpha)^4 - \alpha^2 (-\alpha)^2 + 2(-\alpha) + \alpha + 3 = 0 \]

\[ \Rightarrow \alpha^4 - \alpha^4 - 2\alpha + \alpha + 3 = 0 \]

\[ \Rightarrow -\alpha + 3 = 0 \Rightarrow \alpha = 3 \]

Question 4. Show that $(x - 2)$ is a factor of $3x^3 + x^2 - 20x + 12$.

Solution:

\[ f(x) = 3x^3 + x^2 - 20x + 12 \]

For $(x - 2)$ being a factor of $f(x)$, then $x - 2 = 0 \Rightarrow x = 2$

\[ \therefore f(2) must be zero. \]

Since

\[ f(2) = 3(2)^3 + (2)^2 - 20(2) + 12 \]

\[ = 3(8) + 4 - 40 + 12 \]

\[ = 24 + 4 - 40 + 12 \]

\[ = 40 - 40 = 0 \]

which proves that $(x - 2)$ is a factor of $f(x)$.

TEST YOUR SKILLS

1. Is $(x + 2)$ a factor of $2x^2 + 6x + 4$?
2. Examine whether $(x + 2)$ is a factor of $x^3 + 3x^2 + 5x + 6$.
3. Find the value of $k$, if $(x - 1)$ is a factor of $4x^3 + 3x^2 - 4x + k$.
4. Factorise $6x^2 + 17x + 5$.
5. Using factor theorem, factorise $y^2 - 5y + 6$.
6. Factorise $x^3 - 23x^2 + 142x - 120$.
7. Show that $(x - 2)$ is a factor of $x^3 - 2x^2 - 2x + 4$.
8. For what value of ‘a’, $(x - a)$ is a factor of $x^5 - a^2x^3 + 3x + a - 4$.

**ANSWERS**

1. Yes
2. Yes
3. $k = -3$
4. $(3x + 1)(2x + 5)$
5. $(y - 2)(y - 3)$
6. $(x - 1)(x - 10)(x - 12)$
7. $a = 1$
8. $a = 1$

FACTORISATION USING ALGEBRAIC IDENTITIES

We have used the following identities in our earlier classes:

I. $(x + y)^2 = x^2 + 2xy + y^2$
II. $(x - y)^2 = x^2 - 2xy + y^2$
III. $x^2 - y^2 = (x + y)(x - y)$
IV. $(x + a)(x + b) = x^2 + (a + b)x + ab$

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We shall use these identities to factorise algebraic expressions:

Note: The identity I can be extended to a trinomial such as

\[(x + y + z)^2 = (x + y)^2 + 2(x + y)z + z^2\]
\[= x^2 + y^2 + 2xy + 2xz + 2yz + z^2\]
\[= x^2 + y^2 + z^2 + 2xy + 2yz + 2zx\]

\[\therefore \text{Thus, we have:} \]

**Identity V:** \[(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx\]

We can also extend the identity–I to complete \((x + y)^3\).

We have:

\[(x + y)^3 = (x + y)(x + y)(x + y)\]
\[= (x + y)(x^2 + 2xy + y^2)\]
\[= x(x^2 + 2xy + y^2) + y(x^2 + 2xy + y^2)\]
\[= x^3 + 3x^2y + 3xy^2 + y^3\]
\[= x^3 + y^3 + 3xy(x + y)\]

Thus, we have:

**Identity VI:** \[(x + y)^3 = x^3 + y^3 + 3xy(x + y)\]

By replacing \(y\) by \((-y)\) in identity VI, we have:

**Identity VII:**

\[(x - y)^3 = x^3 - y^3 - 3xy(x - y)\]

or

\[(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3\]

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**NCERT TEXTBOOK QUESTIONS SOLVED**

**EXERCISE 2.5 (Page 48)**

**Question 1.** Use suitable identities to find the following products:

(i) \((x + 4)(x + 10)\)  
(ii) \((x + 8)(x - 10)\)  
(iii) \((3x + 4)(3x - 5)\)

(iv) \(\left(\frac{3}{2}y^2 + 3\right)\left(\frac{3}{2}y^2 - 3\right)\)  
(v) \((3 - 2x)(3 + 2x)\)

**Solution:**

(i) \((x + 4)(x + 10)\):

Using the identity \((x + a)(x + b) = x^2 + (a + b)x + ab\), we have:

\[(x + 4)(x + 10) = x^2 + (4 + 10)x + (4 \times 10)\]
\[= x^2 + 14x + 40\]

(ii) \((x + 8)(x - 10)\):

Here, \(a = 8\) and \(b = (-10)\)

\[\therefore \text{Using \((x + a)(x + b) = x^2 + (a + b)x + ab\), we have:} \]
\[(x + 8)(x - 10) = x^2 + [8 + (-10)]x + [8 \times (-10)]\]
\[= x^2 + [-2]x + [-80]\]
\[= x^2 - 2x - 80\]

(iii) \((3x + 4)(3x - 5)\):

Using the identity \((x + a)(x + b) = x^2 + (a + b)x + ab\), we have

\[(3x + 4)(3x - 5) = (3x)^2 + [4 + (-5)]3x + [4 \times (-5)]\]
\[= 9x^2 + [-1]3x + [-20]\]
\[= 9x^2 - 3x - 20\]

---

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Using the identity \((a + b)(a - b) = a^2 - b^2\), we have:

\[
\left( y^2 + \frac{3}{2} \right) \left( y^2 - \frac{3}{2} \right) = \left[ y^2 \right]^2 - \left[ \frac{3}{2} \right]^2
\]

\[= y^4 - \frac{9}{4} \]

(v) \((3 - 2x)(3 + 2x)\):
Using the identity \((a + b)(a - b) = a^2 - b^2\), we have:
\[(3 - 2x)(3 + 2x) = (3)^2 - (2x)^2 = 9 - 4x^2\]

**Question 2.** Evaluate the following products without multiplying directly:

(i) \(103 \times 107\)  
(ii) \(95 \times 96\)  
(iii) \(104 \times 96\)

**Solution:**

(i) We have

\[103 \times 107 = (100 + 3)(100 + 7)\]

\[= (100)^2 + (3 + 7) \times 100 + (3 \times 7)\]

[Using \((x + a)(x + b) = x^2 + (a + b)x + ab\)]

\[= 10000 + (10) \times 100 + 21\]

\[= 10000 + 1000 + 21\]

\[= 11021\]

(ii) We have

\[95 \times 96 = (100 - 5)(100 - 4)\]

\[= (100)^2 + [(-5) + (-4)] \times 100 + [(-5) \times (-4)]\]

[Using \((x + a)(x + b) = x^2 + (a + b)x + ab\)]

\[= 10000 + [-9] \times 100 + 20\]

\[= 10000 + (-900) + 20\]

\[= 9120\]

(iii) We have

\[104 \times 96 = (100 + 4)(100 - 4)\]

\[= (100)^2 - (4)^2\]  
[Using \((a + b)(a - b) = a^2 - b^2\)]

\[= 10000 - 16\]

\[= 9984\]

**Question 3.** Factorise the following using appropriate identities:

(i) \(9x^2 + 6xy + y^2\)  
(ii) \(4y^2 - 4y + 1\)  
(iii) \(x^2 - \frac{y^2}{100}\)

**Solution:**

(i) We have \(9x^2 + 6xy + y^2\)

\[= (3x)^2 + 2(3x)(y) + (y)^2\]

\[= (3x + y)^2\]  
[Using \(a^2 + 2ab + b^2 = (a + b)^2\)]

\[= (3x + y)(3x + y)\]
(ii) We have \(4y^2 - 4y + 1\)
\[
= (2y)^2 - 2(2y)(1) + (1)^2
\]
\[
= (2y - 1)^2 \quad \because \quad a^2 - 2ab + b^2 = (a - b)^2
\]

(iii) We have \(x^2 - \frac{y^2}{100} = (x)^2 - \left(\frac{y}{10}\right)^2\)
\[
= \left(x + \frac{y}{10}\right)\left(x - \frac{y}{10}\right) \quad [\text{Using } a^2 - b^2 = (a + b)(a - b)]
\]

**Question 4.** Expand each of the following, using suitable identities:

(i) \((x + 2y + 4z)^2\)

(ii) \((2x - y + z)^2\)

(iii) \((-2x + 3y + 2z)^2\)

(iv) \((3a - 7b - c)^2\)

(v) \((-2x + 5y - 3z)^2\)

(vi) \(\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2\)

**Solution:**

(i) \((x + 2y + 4z)^2\)

We have \((x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx\)

\[
\therefore \quad (x + 2y + 4z)^2 = (x)^2 + (2y)^2 + (4z)^2 + 2(x)(2y) + 2(2y)(4z) + 2(4z)(x)
\]
\[
= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8zx
\]

(ii) \((2x - y + z)^2\)

Using \((x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx\), we have

\[
(2x - y + z)^2 = (2x)^2 + (-y)^2 + (z)^2 + 2(2x)(-y) + 2(-y)(z) + 2(z)(2x)
\]
\[
= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4zx
\]

(iii) \((-2x + 3y + 2z)^2\)

Using \((x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx\), we have

\[
(-2x + 3y + 2z)^2 = (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y) + 2(3y)(2z) + 2(2z)(-2x)
\]
\[
= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8zx
\]

(iv) \((3a - 7b - c)^2\)

Using \((x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx\), we have

\[
(3a - 7b - c)^2 = (3a)^2 + (-7b)^2 + (-c)^2 + 2(3a)(-7b) + 2(-7b)(-c) + 2(-c)(3a)
\]
\[
= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ca
\]

(v) \((-2x + 5y - 3z)^2\)

Using \((x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx\), we have

\[
(-2x + 5y - 3z)^2 = (-2x)^2 + (5y)^2 + (-3z)^2 + 2(-2x)(5y) + 2(5y)(-3z) + 2(-3z)(-2x)
\]
\[
= 4x^2 + 25y^2 + 9z^2 + [-20xy] + [-30yz] + [12zx]
\]
\[
= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx
\]

(vi) \(\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2\)

Using \((x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx\), we have

\[
\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2 = \left(\frac{1}{4}a\right)^2 + \left(-\frac{1}{2}b\right)^2 + (1)^2 + 2\left(\frac{1}{4}a\right)\left(-\frac{1}{2}b\right) + 2\left(-\frac{1}{2}b\right)(1) + 2(1)\left(\frac{1}{4}a\right)
\]
\[
\frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 + \left[-\frac{1}{4}ab\right] + [-b] + \left[\frac{1}{2}a\right]
\]

\[
= \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{1}{4}ab - b + \frac{1}{2}a
\]

**Question 5. Factorise:**

(i) \(4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz\)

(ii) \(2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz\)

**Solution:**

(i) \(4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz\)

\[
= (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(-4z)(2x)
\]

\[= (2x + 3y - 4z)^2 \quad \text{[Using Identity V]}\]

\[= (2x + 3y - 4z)(2x + 3y - 4z)\]

(ii) \(2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz\)

\[
= (-\sqrt{2}x)^2 + (y)^2 + (2\sqrt{2}z)^2 + 2(-\sqrt{2}x)(y) + 2(2\sqrt{2}z)(y) + 2(2\sqrt{2}z)(-\sqrt{2}x)
\]

\[= (-\sqrt{2}x + y + 2\sqrt{2}z)^2\]

\[= (-\sqrt{2}x + y + 2\sqrt{2}z)(-\sqrt{2}x + y + 2\sqrt{2}z)\]

**Question 6. Write the following cubes in expanded form:**

(i) \((2x + 1)^3\)

(ii) \((2a - 3b)^3\)

(iii) \(\left[\frac{3}{2}x + 1\right]^3\)

(iv) \(\left[x - \frac{2}{3}y\right]^3\)

**Solution:**

Using Identity VI and Identity VII, we have

\((x + y)^3 = x^3 + y^3 + 3xy(x + y)\), and

\((x - y)^3 = x^3 - y^3 - 3xy(x - y)\).

(i)

\[(2x + 1)^3 = (2x)^3 + (1)^3 + 3(2x)(1)(2x + 1)\]

\[= 8x^3 + 1 + 6x(2x + 1)\] \[\text{[Using Identity VI]}\]

\[= 8x^3 + 1 + 12x^2 + 6x\]

\[= 8x^3 + 12x^2 + 6x + 1\]

(ii)

\[(2a - 3b)^3 = (2a)^3 - (3b)^3 - 3(2a)(3b)(2a - 3b)\]

\[= 8a^3 - 27b^3 - 18ab(2a - 3b)\] \[\text{[Using Identity VII]}\]

\[= 8a^3 - 27b^3 - 36a^2b + 54ab^2\]

\[= 8a^3 - 27b^3 - 36a^2b + 54ab^2\]

(iii)

\[\left[\frac{3}{2}x + 1\right]^3 = \left(\frac{3}{2}x\right)^3 + (1)^3 + 3\left(\frac{3}{2}x\right)(1)\left[\frac{3}{2}x + 1\right]\]

\[= \frac{27}{8}x^3 + 1 + 9x \left[\frac{3}{2}x + 1\right]\] \[\text{[Using Identity VI]}\]

\[= \frac{27}{8}x^3 + 1 + \frac{27}{4}x^2 + \frac{9}{2}x\]

\[= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{18}{2}x + 1\]

(iv)

\[\left[x - \frac{2}{3}y\right]^3 = x^3 - \left(\frac{2}{3}y\right)^3 - 3(x)\left(\frac{2}{3}y\right)\left[x - \frac{2}{3}y\right]\]
Question 7. Evaluate the following using suitable identities:

(i) \( (99)^3 \)  
(ii) \( (102)^3 \)  
(iii) \( (998)^3 \)

Solution:  
(i) \( (99)^3 \)

We have 99 = 100 − 1

\[
99^3 = (100 - 1)^3 = 100^3 - 1^3 - 3(100)(1)(100 - 1) = 1000000 - 1 - 300(100 - 1)
\]

\[
= 1000000 - 1 - 30000 + 300 = 1000300 - 30001 = 970299
\]

(ii) \( (102)^3 \)

We have 102 = 100 + 2

\[
(102)^3 = (100 + 2)^3 = (100^3 + 2^3 + 3(100)(2)[100 + 2])
\]

\[
= 1000000 + 8 + 600[100 + 2] = 1000000 + 8 + 60000 + 1200 = 1061208
\]

(iii) \( (998)^3 \)

We have 998 = 1000 − 2

\[
(999)^3 = (1000 - 2)^3 = (1000^3 - 2^3 - 3(1000)(2)[1000 - 2])
\]

\[
= 1000000000 - 8 - 60000[1000 - 2] = 1000000000 - 8 - 60000000 - 12000 = 994011992
\]

Question 8. Factorise each of the following:

(i) \( 8a^3 + b^3 + 12a^2b + 6ab^2 \)  
(ii) \( 8a^3 - b^3 - 12a^2b + 6ab^2 \)  
(iii) \( 27 - 125a^3 - 135a + 225a^2 \)  
(iv) \( 64a^3 - 27b^3 - 144a^2b + 108ab^2 \)  
(v) \( 27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p \)

Solution:  
(i) \( 8a^3 + b^3 + 12a^2b + 6ab^2 = (2a)^3 + (b)^3 + 6ab(2a + b) = (2a)^3 + (b)^3 + 3(2a)(b)(2a + b) \)
\[ (2a + b)^3 \]  
\[ = (2a + b)(2a + b)(2a + b) \]

(ii) \[ 8a^3 - b^3 - 12a^2b + 6ab^2 = (2a)^3 - (b)^3 - 3(2a)(b)(2a - b) \]
\[ = (2a - b)^3 \]  
\[ = (2a - b)(2a - b)(2a - b) \]

(iii) \[ 27 - 125a^3 - 135a + 225a^2 = (3)^3 - (5a)^3 - 3(3)(5a)[3 - 5a] \]
\[ = (3 - 5a)^3 \]  
\[ = (3 - 5a)(3 - 5a)(3 - 5a) \]

(iv) \[ 64a^3 - 27b^3 - 144a^2b + 108ab^2 = (4a)^3 - (3b)^3 - 3(4a)(3b)[4a - 3b] \]
\[ = (4a - 3b)^3 \]  
\[ = (4a - 3b)(4a - 3b)(4a - 3b) \]

(v) \[ 27p^3 - \frac{9}{2}p^2 + \frac{1}{4}p = (3p)^3 - \bigg(\frac{1}{6}\bigg)^3 - 3(3p)\left[\frac{1}{6}\right]\left[3p - \frac{1}{6}\right] \]
\[ = \left[3p - \frac{1}{6}\right]\left[3p - \frac{1}{6}\right]\left[3p - \frac{1}{6}\right] \]  
\[ = \left(3p - \frac{1}{6}\right)(3p - \frac{1}{6})(3p - \frac{1}{6}) \]

**Question 9.** Verify:

(i) \[ x^3 + y^3 = (x + y)(x^2 - xy + y^2) \]
(ii) \[ x^3 - y^3 = (x - y)(x^2 + xy + y^2) \]

**Solution:**

(i) R.H.S. = \( x(x^2 - xy + y^2) + y(x^2 - xy + y^2) \)
\[ = x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3 \]
\[ = x^3 + y^3 \]
\[ = \text{L.H.S.} \]

(ii) R.H.S. = \( x(x^2 + xy + y^2) - y(x^2 + xy + y^2) \)
\[ = x^3 + x^2y + xy^2 - x^2y - xy^2 - y^3 \]
\[ = x^3 - y^3 \]
\[ = \text{L.H.S.} \]

**Question 10.** Factorise each of the following:

(i) \[ 27y^3 + 125z^3 \]
(ii) \[ 64m^3 - 343n^3 \]

**REMEMBER**

I. \[ x^3 + y^3 = (x + y)(x^2 - xy + y^2) \]

II. \[ x^3 - y^3 = (x - y)(x^2 + xy + y^2) \]

**Solution:**

(i) Using the identity \((x^3 + y^3) = (x + y)(x^2 - xy + y^2)\), we have
\[ 27y^3 + 125z^3 = (3y)^3 + (5z)^3 \]
\[ = (3y + 5z)[(3y)^2 - (3y)(5z) + (5z)^2] \]
\[ = (3y + 5z)(9y^2 - 15yz + 25z^2) \]

(ii) Using the identity \((x^3 - y^3) = (x - y)(x^2 + xy + y^2)\), we have
\[ 64m^3 - 343n^3 = (4m)^3 - (7n)^3 \]
\[ = (4m - 7n)[(4m)^2 + (4m)(7n) + (7n)^2] \]
\[ = (4m - 7n)(16m^2 + 28mn + 49n^2) \]
Question 11. Factorise $27x^3 + y^3 + z^3 - 9xyz$.

Solution:

REMEMBER

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

We have

$$27x^3 + y^3 + z^3 - 9xyz = (3x)^3 + (y)^3 + (z)^3 - 3(3x)(y)(z)$$

Using the identity $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$, we have

$$(3x)^3 + (y)^3 + (z)^3 - 3(3x)(y)(z)$$

$$= (3x + y + z)[(3x)^2 + y^2 + z^2 - (3x \times y) - (y \times z) - (z \times 3x)]$$

$$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3zx)$$

Question 12. Verify that $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)(x^2 + y^2 + y^2 + z^2 + z^2 + x^2 - 2xy - 2yz - 2zx)$

Solution: R.H.S. = $\frac{1}{2}(x + y + z)((x - y)^2 + (y - z)^2 + (z - x)^2]$

$$= \frac{1}{2}(x + y + z)[(x^2 + y^2 - 2xy) + (y^2 + z^2 - 2yz) + (z^2 + x^2 - 2zx)]$$

$$= \frac{1}{2}(x + y + z)[x^2 + y^2 + y^2 + z^2 + z^2 + x^2 - 2xy - 2yz - 2zx]$$

$$= \frac{1}{2}(x + y + z)[2(x^2 + y^2 + z^2 - xy - yz - zx)]$$

$$= 2 \times \frac{1}{2} \times (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

$$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

$$= x^3 + y^3 + z^3 - 3xyz$$

L.H.S.

Question 13. If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$.

Solution: Since

$$x + y + z = 0$$

∴

$$x + y = -z$$

or

$$(x + y)^3 = (-z)^3$$

or

$$x^3 + y^3 + 3xy(x + y) = -z^3$$

or

$$x^3 + y^3 + 3xy(-z) = -z^3$$

[∵ \(x + y = (-z)\)]

or

$$x^3 + y^3 = 3xyz$$

or

$$(x^3 + y^3 + z^3) - 3xyz = 0$$

or

$$x^3 + y^3 + z^3 = 3xyz$$

Hence, if $x + y + z = 0$, then $(x^3 + y^3 + z^3) = 3xyz$.

Question 14. Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3 + (7)^3 + (3)^3$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Solution: (i) $(-12)^3 + (7)^3 + (3)^3$

Let \(x = -12, y = 7\), and \(z = 5\)
Then \( x + y + z = -12 + 7 + 5 = 0 \)

We know that if \( x + y + z = 0 \), then \( x^3 + y^3 + z^3 = 3xyz \).

\[
(-12)^3 + (7)^3 + (5)^3 = 3 \times (-12)(7)(5) \\
= 3 \times -420 \\
= -1260
\]

Thus, \((-12)^3 + (7)^3 + (5)^3 = -1260\)

(ii) \((28)^3 + (-15)^3 + (-13)^3\)

Let \( x = 28, y = -15 \) and \( z = -13 \)

\[
x + y + z = 28 - 15 - 13 = 0
\]

We know that if \( x + y + z = 0 \), then \( x^3 + y^3 + z^3 = 3xyz \).

\[
(28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13) \\
= 3(5460) \\
= 16380
\]

Thus, \((28)^3 + (-15)^3 + (-13)^3 = 16380\)

**Question 15.** Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

**Area: 25a^2 – 35a + 12**

**Area: 35y^2 + 13y – 12**

**REMEMBER**

Area of a rectangle = (Length) \(\times\) (Breadth)

**Solution:**

(i) Area = \(25a^2 – 35a + 12\)

We have to factorise the polynomial: \(25a^2 – 35a + 12\)

Splitting the co-efficient of \(a\), we have

\[ -35 = (-20) + (-15) \quad \Rightarrow \quad 25 \times 12 = 300 \text{ and } (-20) \times (-15) = 300 \]

\[ 25a^2 – 35a + 12 = 25a^2 – 20a – 15a + 12 \]

\[ = 5a(5a – 4) – 3(5a – 4) \]

\[ = (5a – 4)(5a – 3) \]

Thus, the possible length and breadth are \((5a – 3)\) and \((5a – 4)\).

(ii) Area = \(35y^2 + 13y – 12\)

We have to factorise the polynomial \(35y^2 + 13y – 12\).

Splitting the middle term, we get

\[ 13y = 28y – 15y \quad \Rightarrow \quad 28 \times (-15) = -420 \text{ and } -12 \times 35 = -420 \]

\[ 35y^2 + 13y – 12 = 35y^2 + 28y – 15y – 12 \]

\[ = 7y(5y + 4) – 3(5y + 4) \]

\[ = (5y + 4)(7y – 3) \]

Thus, the possible length and breadth are \((7y – 3)\) and \((5y + 4)\).

**Question 16.** What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

Volume: \(3x^2 – 12x\)

Volume: \(12ky^2 + 8ky – 20k\)
REMEMBER
Volume of a cuboid = (Length) \times (Breadth) \times (Height)

Solution:
(i) Volume = 3x^2 – 12x
On factorising 3x^2 – 12x, we have
\[3x^2 – 12x = 3[x^2 – 4x]\]
\[= 3 \times [x(x – 4)]\]
\[= 3 \times x \times (x – 4)\]
\[\therefore\] The possible dimensions of the cuboid are: 3, x and (x – 4) units.

(ii) Volume = 12ky^2 + 8ky – 20k
We have 12ky^2 + 8ky – 20k = 4[3ky^2 + 2ky – 5k]
\[= 4[k(3y^2 + 2y – 5)]\]
\[= 4 \times k \times (3y^2 + 2y – 5)\]
\[= 4k[3y^2 – 3y + 5y – 5]\] (Splitting the middle term)
\[= 4k[3y(y – 1) + 5(y – 1)]\]
\[= 4k[(3y + 5)(y – 1)]\]
\[= 4k \times (3y + 5) \times (y – 1)\]
Thus, the possible dimensions are: 4k, (3y + 5) and (y – 1) units.

ADDITIONAL QUESTIONS SOLVED

Question 1. Factorise the polynomial \(x^2 + \frac{1}{x^2} – 2x – \frac{2}{x} + 2\).

Solution: We have \(x^2 + \frac{1}{x^2} – 2x – \frac{2}{x} + 2\)
\[= \left[x^2 + \frac{1}{x^2} + 2\right] – 2\left[x + \frac{1}{x}\right]\]
\[= \left[x^2 + \frac{1}{x^2} + 2\right] – 2\left[x + \frac{1}{x}\right]\]
\[= \left[x + \frac{1}{x}\right]^2 – 2\left[x + \frac{1}{x}\right]\]
\[= \left(x + \frac{1}{x}\right)\left(x + \frac{1}{x}\right) – 2\left(x + \frac{1}{x}\right)\]

Question 2. Factorise \(a(a – 1) – b(b – 1)\).

Solution: We have \(a(a – 1) – b(b – 1)\)
\[= a^2 – a – b^2 + b\]
\[= a^2 – b^2 – (a – b)\] (Rearranging the terms)
\[= [(a + b)(a – b)] – (a – b)\] \[\therefore\] \(x^2 – y^2 = (x + y)(x – y)\)
= (a – b)((a + b) – 1)
= (a – b)(a + b – 1)
Thus, \(a(a – b) - b(b – 1) = (a – b)(a + b – 1)\)

**Question 3.** Show that \(x^3 + y^3 = (x + y)(x^2 – xy + y^2)\).

**Solution:** Since \((x + y)^3 = x^3 + y^3 + 3xy(x + y)\)
\[
\therefore \quad x^3 + y^3 = [(x + y)^3] – 3xy(x + y) = (x + y)(x + y)^2 – 3xy] = (x + y)[x^2 + y^2 – xy] = \frac{25}{4} x^2 – \frac{y^2}{9}
\]
Thus, \(x^3 + y^3 = (x + y)(x^2 – xy + y^2)\)

**Question 4.** Show that \(x^3 – y^3 = (x – y)(x^2 + xy + y^2)\).

**Solution:** Since \((x – y)^3 = x^3 – y^3 – 3xy(x – y)\)
\[
\therefore \quad x^3 – y^3 = [(x – y)^3] + 3xy(x – y) = (x – y)(x – y)^2 + 3xy] = (x – y)[x^2 + y^2 – 2xy + 3xy] = (x – y)(x^2 + y^2 + xy) = \frac{25}{4} x^2 – \frac{y^2}{9}
\]
Thus, \(x^3 – y^3 = (x – y)(x^2 + xy + y^2)\)

**TEST YOUR SKILLS**

1. Using appropriate identities, find the following products:
   (i) \((x + 6)(x + 6)\)  
   (ii) \((x + 5)(x – 6)\)

2. Without multiplying, find the product of:
   (i) \(105 \times 106\)  
   (ii) \(98 \times 99\)

3. Factorise:
   (i) \(49a^2 + 70ab + 25b^2\)  
   (ii) \(\frac{25}{4} x^2 – \frac{y^2}{9}\)

4. Expand: \([3a + 4b + 5c]^2\)

5. Expand: \([4a – 2b – 3c]^2\)

6. Factorise: \(4x^2 + y^2 + z^2 – 4xy – 2yz + 4xz\)

7. Expand:
   (i) \((3a + 4b)^3\)  
   (ii) \((5p – 3q)^3\)

8. Using suitable identities, evaluate:
   (i) \((103)^3\)  
   (ii) \((999)^3\)
9. Factorise: \(8x^3 + 27y^3 + 36x^2y + 54xy^2\)

10. Factorise: \(8x^3 + y^3 + 27z^3 - 18xyz\)

11. Factorise: \((2x + 3y)^3 - (2x - 3y)^3\)

12. Factorise: \(a^3 - b^3 + 1 + 3ab\)

13. Factorise: \((p - q)^3 + (q - r)^3 + (r - p)^3\)

14. What is the degree of the polynomial: \((x - 1) (x + 1) (x - 2)\)?

15. Find the value of the polynomial: \(7y - 4y^2 + 5\) at \(y = -1\)

16. When the polynomials \(6x^3 + 11x^2 - 12x - a\) and \(3x^3 + x^2 - 12x - 2\) are divided by \((3x + 1)\) then remainder is same. Find the value of ‘\(a\)’.

\[
\text{Hint: } F(x) = 6x^3 + 11x^2 - 12x - a \Rightarrow F\left(-\frac{1}{3}\right) = 5 - a \quad \left[\because 3x+1 = 0 \Rightarrow x = -\frac{1}{3}\right]
\]

\[f(x) = 3x^3 + x^2 - 12x - 2 \Rightarrow f\left(-\frac{1}{3}\right) = 2\]

\[\therefore 5 - a = 2 \Rightarrow a = 3\]

17. If one zero of \(4x^3 - 20x^2 + 27x - 9\) is 3, find the other zeroes of the polynomial:

\[
\text{Hint: } 3 \text{ is zero of } f(x) \Rightarrow (x - 3) \text{ is a factor of } f(x)
\]

\[\therefore f(x) + (x - 3) \Rightarrow 4x^2 - 8x + 3 = (2x - 1) (2x - 3)
\]

\[\Rightarrow 2x - 1 = 0 \text{ gives } x = \frac{1}{2} \text{ and } 2x - 3 = 0 \text{ gives } x = \frac{3}{2}\]

\[\therefore \text{ other zeroes are } \frac{1}{2} \text{ and } \frac{3}{2}\]

18. If the polynomials \(ax^3 + 3x^2 - 13\) and \(2x^3 - 5x + a\) are divided by \((x - 2)\), the remainder is same. Find the value of \(a\).

19. If \(x + y = 5\) and \(xy = 4\), find \(x^3 + y^3\)

20. Factorise: \(27a^3 - 64b^3 - 108a^2b + 144ab^2 + 9a - 12b\)

\[
\text{Hint: } 27a^3 - 64b^3 - 108a^2b + 144ab^2 + 9a - 12b
\]

\[= (3a)^3 - (4b)^3 - 3(3a)^2 (4b) + 3(3a)(4b)^2 + 3(3a - 4b)
\]

\[= (3a - 4b)^3 + 3(3a - 4b) = (3a - 4b) [(3a - 4b)^2 + 3]\]
6. \((2x - y + z)(2x - y + z)\)
7. (i) \(27a^3 + 64b^3 + 108a^2b + 144ab^2\)
   (ii) \(125p^3 - 27p^3 - 225p^2q + 135pq^2\)
8. (i) \(1092727\)
   (ii) \(997002999\)
9. \((2x + 3y)(2x + 3y)(2x + 3y)\)
10. \((2x + y + 3z)(4x^2 + y^2 + 9z^2 - 2xy - 3yz - 6xz)\)
11. \(18y(4x^2 + 3y^2)\)
12. \((a - b + 1)(a^2 + b^2 + ab - a + b + 1)\)
13. \(3(p - q)(q - r)(r - p)\)
14. \(3\)
15. \(-6\)
16. \(a = 3\)
17. \(\frac{1}{2}\) and \(\frac{3}{2}\)
18. \(a = 1\)
19. \(65\)
20. \((3a - 4b)([3a - 4b]^2 + 3]\)

**ADDITIONAL QUESTIONS SOLVED**

I. VERY SHORT ANSWER TYPE QUESTIONS (Carrying 1 Mark)

**Question 1.** What is \(p(-2)\) for the polynomial \(p(t) = t^2 - t + 1\)?

**Solution:**
\[
p(t) = t^2 - t + 1
\]
\[
p(-2) = (-2)^2 - (-2) + 1
\]
\[
= 4 + 2 + 1
\]
\[
= 7
\]

**Question 2.** If \(x - \frac{1}{x} = -1\) then what is \(x^2 + \frac{1}{x^2}\)?

**Solution:**
\[
\left( x - \frac{1}{x} \right)^2 = x^2 + \frac{1}{x^2} - 2(x \left( \frac{1}{x} \right))
\]
\[
\Rightarrow (1)^2 = x^2 + \frac{1}{x^2} - 2
\]
\[
\Rightarrow 1 = x^2 + \frac{1}{x^2} - 2
\]
\[
\Rightarrow x^2 + \frac{1}{x^2} = 1 + 2 = 3
\]

**Question 3.** If \(x + y = -1\), then what is the value of \(x^3 + y^3 - 3xy\)?

**Solution:**
We have \(x^3 + y^3 = (x + y)(x^2 - xy + y^2)\)
\[
\Rightarrow x^3 + y^3 = (-1)(x^2 + 2xy + y^2) + 3xy
\]
\[
\Rightarrow x^3 + y^3 = -1(x^2 + y^2) - 3xy
\]
\[
\Rightarrow x^3 + y^3 - 3xy = -1(-1)^2 = -1(1)
\]
\[
\Rightarrow x^3 + y^3 - 3xy = -1
\]
**Question 4.** Show that \( p(x) \) is not a multiple of \( g(x) \), when
\[
p(x) = x^3 + x - 1 \\
g(x) = 3x - 1
\]
Solution: \( g(x) = 3x - 1 = 0 \Rightarrow x = \frac{1}{3} \)

\[
\therefore \quad \text{Remainder} = p\left(\frac{1}{3}\right)
\]
\[
= \left(\frac{1}{3}\right)^3 + \frac{1}{3} - 1
\]
\[
= \frac{1}{27} + \frac{1}{3} - 1
\]
\[
= \frac{1 + 9 - 27}{27} = \frac{-17}{27}
\]

Since remainder \( \neq 0 \), so \( p(x) \) is not a multiple of \( g(x) \).

**Question 5.** (a) Find the value of \( a \) if \( x - a \) is a factor of \( x^3 - ax^2 + 2x + a - 5 \).

(b) Find the value of \( a \), if \( (x - a) \) is a factor of \( x^3 + ax^2 + 2x + a - 1 \)

(c) If \( x + 1 \) is a factor of \( ax^3 + x^2 - 2x + 4a - 9 \) find the value of \( a \).

Solution: (a) Let \( p(x) = x^3 - ax^2 + 2x + a - 5 \)

since \( x - a \) is a factor of \( p(x) \), so \( p(a) = 0 \)

\( \Rightarrow \quad (a)^3 - a(a)^2 + 2(a) + a - 5 = 0 \)

\( \Rightarrow \quad 3a - 5 = 0 \Rightarrow a = \frac{5}{3} \)

(b) Here, \( p(x) = x^3 - ax^2 + 2x + a - 1 \)

\( \therefore \quad x - a \) is a factor of \( p(x) \)

\( \therefore \quad p(a) = 0 \)

\( \Rightarrow \quad a^3 - a(a)^2 + 2(a) + a - 1 = 0 \)

\( \Rightarrow \quad a^3 - a^3 + 2a + a - 1 = 0 \)

\( \Rightarrow \quad 3a - 1 = 0 \)

\( \Rightarrow \quad a = \frac{1}{3} \)

(c) Here, \( x + 1 \) is a factor of \( p(x) = ax^3 + x^2 - 2x + 4a - 9 \)

\( \therefore \quad p(-1) = 0 \)

\( \Rightarrow \quad a(-1)^3 + (-1)^2 - 2(-1) + 4a - 9 = 0 \)

\( \Rightarrow \quad -a + 1 + 2 + 4a - 9 = 0 \)

\( \Rightarrow \quad 3a - 6 = 0 \Rightarrow a = 2 \)

**Question 6.** Without finding the cubes factorise \( (a - b)^3 + (b - c)^3 + (c - a)^3 \)

Solution: If \( x + y + z = 0 \) then \( x^3 + y^3 + z^3 = 3xyz \)

Here,
\[
(a - b) + (b - c) + (c - a) = 0
\]

\( \therefore \quad (a - b)^3 + (b - c)^3 + (c - a)^3 = 3(a - b)(b - c)(c - a) \)
Question 7. What is zero of a non-zero constant polynomial?
Solution: The zero of a non-zero constant polynomial is ‘0’.

Question 8. What is the coefficient of a zero polynomial?
Solution: The coefficient of a zero polynomial is 1.

Question 9. What is the degree of a biquadratic polynomial?
Solution: ∵ The degree of quadratic polynomial is 2.
∴ The degree of biquadratic polynomial is 4.

Question 10. Is the statement: ‘0’ may be a zero of polynomial, true?
Solution: Yes, this statement is true.

Question 11. What is the value of \((x + a) (x + b)\)?
Solution: The value of \((x + a) (x + b) = x^2 + (a + b) x + ab\).

Question 12. What is the value of \((x + y + z)^2 - 2[xy + yz + zx]\)?
Solution: ∵ \((x + y + z)^2 = x^2 + y^2 + z^2 + 2 xy + 2 yz + 2 zx\)
∴ \((x + y + z)^2 - 2[xy + yz + zx] = x^2 + y^2 + z^2\)

Question 13. What is the value of \((x + y)^3 - 3xy (x + y)\)?
Solution: ∵ \((x + y)^3 = x^3 + y^3 + 3xy (x + y)\)
∴ \([x^3 + y^3 + 3xy (x + y)] - [3xy (x + y)] = x^3 + y^3\)
⇒ \((x + y)^3 - 3xy(x + y) = x^3 + y^3\)
Thus, value of \(x^3 + y^3\) is \((x + y)^3 - 3xy (x + y)\)

Question 14. Write the value of \(x^3 - y^3\).
Solution: The value of \(x^3 - y^3\) is \((x - y)^3 + 3xy (x - y)\)

Question 15. Write the degree of the polynomial \(4x^4 + ox^3 + ox^5 + 5x + 7\)?
Solution: The degree of \(4x^4 + 0x^3 + 0x^5 + 5x + 7\) is 4.

Question 16. What is the zero of the polynomial \(p(x) = 2x + 5\)?
Solution: ∴ \(p(x) = 0 \Rightarrow 2x + 5 = 0 \Rightarrow x = \frac{-5}{2}\)
∴ zero of \(2x + 5\) is \(\frac{-5}{2}\)

Question 17. Which of the following is one of the zero of the polynomial \(2x^2 + 7x - 4\)?
\(2, \frac{1}{2}, -\frac{1}{2}, -2\ ?\)
Solution: ∴ \(2x^2 + 7x - 4 = 2x^2 + 8x - x - 4\)
\Rightarrow 2x (x + 4) - 1 (x + 4) = 0
\Rightarrow (x + 4) (2x - 1) = 0 \Rightarrow x = -4, x = \frac{1}{2}\)
∴ One of the zero of \(2x^2 + 7x - 4\) is \(\frac{1}{2}\)
**Question 18.** If \( a + b + 2 = 0 \), then what is the value of \( a^3 + b^3 + 8 \).

**Solution:** 
\[
\therefore \quad x + y + z = 0 \Rightarrow x^3 + y^3 + z^3 = 3xyz
\]
\[
\therefore \quad a + b + 2 = 0 \Rightarrow (a)^3 + (b)^3 + (2)^3 = 3(a \times b \times 2) = 6ab
\]
\[
\Rightarrow \text{The value of } a^3 + b^3 + 8 \text{ is } 6ab
\]

**Question 19.** If \( 49x^2 - p = \left( \frac{7}{3}x - \frac{1}{3} \right) \left( \frac{7}{3}x - \frac{1}{3} \right) \), what is the value of \( p \)?

**Solution:** 
\[
\therefore \quad \left( \frac{7}{3}x - \frac{1}{3} \right) \left( \frac{7}{3}x - \frac{1}{3} \right) = \left( \frac{7}{3}x \right)^2 - \left( \frac{1}{3} \right)^2 = 49x^2 - \frac{1}{9}
\]
\[
\therefore \quad 49x^2 - p = 49x^2 - \frac{1}{9} \Rightarrow p = \frac{1}{9}
\]

**Question 20.** If \( x + \frac{1}{x} = 1 \), then what is the value of \( x^2 + \frac{1}{x^2} \)?

**Solution:** 
\[
\therefore \quad (x + \frac{1}{x})^2 = 1^2 \Rightarrow x^2 + \frac{1}{x^2} + 2 = 1
\]
\[
\Rightarrow x^2 + \frac{1}{x^2} = 1 - 2
\]
\[
\Rightarrow x^2 + \frac{1}{x^2} = -1
\]

II. SHORT ANSWER TYPE QUESTIONS (Carrying 2 Marks)

**Question 1.** Write the numerical co-efficient and degree of each term of: \( \frac{x}{2} - 3x^2 + \frac{5}{2}x^3 - 5x^4 \)

**Solution:**
<table>
<thead>
<tr>
<th>Term</th>
<th>Numerical co-efficient</th>
<th>Degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{x}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>1</td>
</tr>
<tr>
<td>(-3x^2)</td>
<td>-3</td>
<td>2</td>
</tr>
<tr>
<td>( \frac{5}{2}x^3 )</td>
<td>( \frac{5}{2} )</td>
<td>3</td>
</tr>
<tr>
<td>(-5x^4)</td>
<td>-5</td>
<td>4</td>
</tr>
</tbody>
</table>

**Question 2.** Find the remainder when \( x^3 - ax^2 + 4x - a \) is divided by \( (x - a) \).

**Solution:**
\[
p(x) = x^3 - ax^2 + 4x - a \]
\[
(x - a) = 0 \quad \Rightarrow \quad x = a
\]
\[
\therefore \quad p(a) = (a)^3 - a(a)^2 + 4(a) - a
\]
\[
\quad = a^3 - a^3 + 4a - a
\]
\[
\quad = 4a - a
\]
\[
\quad = 3a
\]
\[
\therefore \quad \text{The required remainder} = 3a
\]
**Question 3.** When the polynomial $kx^3 + 9x^2 + 4x - 8$ is divided $x + 3$, then a remainder 7 is obtained. Find the value of $k$.

**Solution:** Here, $p(x) = kx^3 + 9x^2 + 4x - 8$

Since, Divisor = $x + 3$

\[ x + 3 = 0 \Rightarrow x = -3 \]

\[ p(-3) = 7 \]

\[ k(-3)^3 + 9(-3)^2 + 4(-3) - 8 = 7 \]

\[ -27k + 81 - 12 - 8 = 7 \]

\[ -27k + 71 = 27 + 81 + 8 \]

\[ -27k = 27 - 81 \]

\[ -27k = -54 \]

\[ k = \frac{-54}{-27} = 2 \]

Thus, the required value of $k = 2$.

**Question 4.**

(a) For what value of $k$, the polynomial $x^2 + (4 - k)x + 2$ is divisible by $x - 2$?

(b) For what value of ‘$m$’ is $x^3 - 2mx^2 + 16$ is divisible by $(x + 2)$?

**Solution:** (a) Here $p(x) = x^2 + 4x - kx + 2$

If $p(x)$ is exactly divisible by $x - 2$, then $p(2) = 0$

i.e. \[ (2)^2 + 4(2) - k(2) + 2 = 0 \]

\[ 4 + 8 - 2k + 2 = 0 \]

\[ 14 - 2k = 0 \]

\[ 2k = 14 \]

\[ k = \frac{14}{2} = 7 \]

Thus, the required value of $k$ is 7.

(b) Here, \[ p(x) = x^3 - 2mx^2 + 16 \]

\[ p(-2) = (-2)^3 - 2(-2)^2m + 16 \]

\[ = -8 - 8m + 16 \]

Since, $p(x)$ is divisible by $x + 2$

\[ p(-2) = 0 \]

or \[ -8m + 8 = 0 \]

\[ m = 1 \]

**Question 5.** Factorise $x^2 - x - 12$.

**Solution:** We have $x^2 - x - 12 = x^2 - 4x + 3x - 12$

\[ = x(x - 4) + 3(x - 4) \]

\[ = (x - 4)(x + 3) \]

Thus, $x^2 - x - 12 = (x - 4)(x + 3)$
Question 6. If \( x + \frac{1}{2x} = 5 \), then find the value of \( x^2 + \frac{1}{4x^2} \).

Solution: We have \( x + \frac{1}{2x} = 5 \)

Squaring both sides, we get

\[
\left( x + \frac{1}{2x} \right)^2 = 5^2
\]

\[
\Rightarrow x^2 + \frac{1}{4x^2} + 2 \times x \times \frac{1}{2x} = 25
\]

\[
\Rightarrow x^2 + \frac{1}{4x^2} + 1 = 25
\]

\[
\Rightarrow x^2 + \frac{1}{4x^2} = 25 - 1 = 24
\]

Thus, the required value of \( x^2 + \frac{1}{4x^2} \) is 24.

III. SHORT ANSWER TYPE QUESTIONS (Carrying 3 Marks)

Question 1. Check whether \((x – 1)\) is a factor of the polynomial \(x^3 – 27x^2 + 8x + 18\).

Solution: Here, \( p(x) = x^3 – 27x^2 + 8x + 18 \)

\((x – 1)\) will be a factor of \( p(x) \) only if \((x – 1)\) divides \( p(x) \) leaving a remainder 0.

For \( x – 1 = 0 \) \( \Rightarrow \) \( x = 1 \)

\[
p(1) = (1)^3 – 27(1)^2 + 8(1) + 18
\]

\[
= 1 - 27 + 8 + 18
\]

\[
= 27 - 27
\]

\[
= 0
\]

Since, \( p(1) = 0 \)

\( \therefore \) \((x – 1)\) is a factor of \( p(x) \).

Thus, \((x – 1)\) is a factor of \(x^3 – 27x^2 + 8x + 18\).

Question 2. Find the remainder when \( f(x) = 3x^4 + 2x^3 – \frac{x^2}{3} – \frac{x}{9} + \frac{12}{27} \) is divided by \( x + \frac{2}{3} \).

Solution: Here \( f(x) = 3x^4 + 2x^3 – \frac{x^2}{3} – \frac{x}{9} + \frac{12}{27} \)

\[
\text{Divisor} = x + \frac{2}{3}
\]

Since,

\[
x + \frac{2}{3} = 0 \Rightarrow x = -\frac{2}{3}
\]

\( \therefore \) \( \text{Remainder} = f \left( -\frac{2}{3} \right) \)
i.e. \[ \text{Remainder} = 3 \left( \frac{-2}{3} \right)^4 + 2 \left( \frac{-2}{3} \right)^3 - \frac{1}{3} \left( \frac{-2}{3} \right)^2 - \frac{1}{9} \left( \frac{-2}{3} \right) + \frac{12}{27} \]

\[ = 3 \left( \frac{16}{81} \right) + 2 \left( \frac{-8}{27} \right) - \frac{1}{3} \left( \frac{4}{9} \right) - \frac{1}{9} \left( \frac{-2}{3} \right) + \frac{12}{27} \]

\[ = \frac{16}{27} - \frac{16}{27} - \frac{4}{27} + \frac{2}{27} + \frac{12}{27} \]

\[ = \frac{16 + 2 + 12}{27} - \frac{16 + 4}{27} \]

\[ = \frac{30}{27} - \frac{20}{27} = \frac{10}{27} \]

Thus, the required remainder \( = \frac{10}{27} \).

**Question 3.** Find the value of \( k \), if \( (x - k) \) is a factor of \( x^6 - kx^5 + x^4 - kx^3 + 3x - k + 4 \).

**Solution:** Here \( p(x) = x^6 - kx^5 + x^4 - kx^3 + 3x - k + 4 \)

If \( (x - k) \) is a factor of \( p(x) \), then \( p(k) = 0 \)

i.e. \( (k)^6 - k(k^5) + k^4 - k(k^3) + 3k - k + 4 = 0 \)

or \( k^6 - k^6 + k^4 - k^4 + 3k - k + 4 = 0 \)

or \( 2k + 4 = 0 \)

or \( 2k = -4 \)

or \( k = \frac{-4}{2} = -2 \)

Thus, the required value of \( k \) is \(-2\).

**Question 4.** Factorise: \( 9a^2 - 9b^2 + 6a + 1 \)

**Solution:** \( 9a^2 - 9b^2 + 6a + 1 = [9a^2 + 6a + 1] - 9b^2 \)

\[ = [(3a)^2 + 2(3a)(1) + (1)^2] - (3b)^2 \]

\[ = (3a + 1)^2 - (3b)^2 \]

\[ = [(3a + 1) + 3b][(3a + 1) - 3b] \]

[using \( x^2 - y^2 = (x - y)(x + y) \)]

\[ = (3a + 1 + 3b)(3a + 1 - 3b) \]

**Question 5.** Find the value of \( x^3 + y^3 - 12xy + 64 \), when \( x + y = -4 \).

**Solution:** \( x^3 + y^3 - 12xy + 64 = (x)^3 + (y)^3 + (4)^3 - 3(x)(y)(4) \)

\[ = [x^2 + y^2 + 4^2 - xy - y \cdot 4 - 4 \cdot x](x + y + 4) \]

\[ = [x^2 + y^2 + 16 - xy - 4y - 4x][x + y + 4] \] \( \ldots \) (1)

Since, \( x + y = -4 \)

\[ \therefore \quad x + y + 4 = 0 \] \( \ldots \) (2)
From (1) and (2), we have
\[ x^3 + y^3 - 12xy + 64 = [x^2 + y^2 + 16 - xy - 4y - 4x][0] = 0 \]
Thus,
\[ x^3 + y^3 - 12xy + 64 = 0. \]

IV. LONG ANSWER TYPE QUESTIONS (Carrying 4 Marks)

**Question 1.** Factorise: \( x^2 - \frac{5}{12} x + \frac{1}{24} \)

**Solution:**
\[
x^2 - \frac{5}{12} x + \frac{1}{24} = \left[ 24(x^2) - 24 \left( \frac{5}{12} x \right) + (24) \frac{1}{24} \right] \times \frac{1}{24}
\]
\[
= [24x^2 - 10x + 1] \times \frac{1}{24}
\]
\[
= [24x^2 - 6x - 4x + 1] \times \frac{1}{24}
\]
\[
= [(4x - 1)(6x - 1)] \times \frac{1}{24}
\]
Thus,
\[ x^2 - \frac{5}{12} x + \frac{1}{24} = \frac{1}{24} (4x - 1)(6x - 1) \]

**Question 2.** Factorise: \( (x^6 - y^6) \)

**Solution:**
\[
x^6 - y^6 = (x^3)^2 - (y^3)^2
\]
\[
= (x^3 - y^3)(x^3 + y^3) \quad [\because a^2 - b^2 = (a + b)(a - b)]
\]
\[
= [(x - y)(x^2 + xy + y^2)][(x + y)(x^2 - xy + y^2)]
\]
\[
[\because a^3 + b^3 = (a^2 + b^2 - ab)(a + b) \text{ and } a^3 - b^3 = (a^2 + ab + b^2)(a - b)]
\]
\[
= (x - y)(x + y)(x^2 + xy + y^2)(x^2 - xy + y^2)
\]
Thus,
\[ x^6 - y^6 = (x - y)(x + y)(x^2 + xy + y^2)(x^2 - xy + y^2) \]

**Question 3.** If the polynomials \( 2x^3 + 3x^2 - a \) and \( ax^3 - 5x + 2 \) leave the same remainder when each is divided by \( x - 2 \), find the value of ‘\( a \)’.

**Solution:** Let \( p(x) = 2x^3 + 3x^2 - a \) and \( f(x) = ax^3 - 5x + 2 \)

<table>
<thead>
<tr>
<th>When ( p(x) ) is divided by ( x - 2 ) then remainder = ( p(2) )</th>
<th>When ( f(x) ) is divided by ( x - 2 ) then remainder = ( f(2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>since ( p(2) = 2(2)^3 + 3(2)^2 - a )</td>
<td>since, ( f(2) = a(2)^3 - 5(2) + 2 )</td>
</tr>
<tr>
<td>= 2(8) + 3(4) - a</td>
<td>= ( a(8) - 10 + 2 )</td>
</tr>
<tr>
<td>= 16 + 12 - a</td>
<td>= 8a - 8</td>
</tr>
<tr>
<td>( \therefore ) Remainder = 28 - a</td>
<td>( \therefore ) Remainder = 8a - 8</td>
</tr>
</tbody>
</table>
since, \( p(x) \) and \( f(x) \) leave the same remainder

\[
\begin{align*}
\therefore & \quad 28 - a = 8a - 8 \\
\Rightarrow & \quad 8a + a = 28 + 8 \\
\Rightarrow & \quad 9a = 36 \\
\Rightarrow & \quad a = \frac{36}{9} = 4
\end{align*}
\]

Thus, \( a = 4 \)

**Question 4.** Find the values of ‘\( p \)’ and ‘\( q \)’, so that \((x - 1)\) and \((x + 2)\) are the factors of \( x^3 + 10x^2 + px + q \).

**Solution:** Here \( f(x) = x^3 + 10x^2 + px + q \)

Since, \( x + 2 = 0 \) \[∵ \quad x + 2 \text{ is a factor of } f(x) \]

\[
\begin{align*}
\therefore & \quad x = -2 \\
\text{i.e.} & \quad \left(-2\right)^3 + 10(-2)^2 + p(-2) + q = 0 \quad \text{[Factor theorem]} \\
\Rightarrow & \quad -8 + 40 - 2p + q = 0 \\
\Rightarrow & \quad 32 - 2p + q = 0 \\
\Rightarrow & \quad 2p - q = 32 \quad \cdots(1)
\end{align*}
\]

Also \( x - 1 = 0 \) \( \Rightarrow \quad x = 1 \)

If \((x - 1)\) is a factor of \( f(x) \), then \( f(1) \) must be equal to 0. \[\text{[Factor theorem]} \]

\[
\begin{align*}
\text{i.e.} & \quad (1)^3 + 10(1)^2 + p(1) + q = 0 \\
\Rightarrow & \quad 1 + 10 + p + q = 0 \\
\Rightarrow & \quad 11 + p + q = 0 \\
\Rightarrow & \quad p + q = -11 \quad \cdots(2)
\end{align*}
\]

Now, adding (1) and (2), we get

\[
\begin{align*}
3p - q &= 32 \\
p + q &= -11
\end{align*}
\]

\[
\Rightarrow \quad 4p = 21
\]

\[
\Rightarrow \quad p = \frac{21}{4} = 7
\]

Now we put \( p = 7 \) in (2), we have

\[
7 + q = -11
\]

\[
\Rightarrow \quad q = -11 - 7 = -18
\]

Thus, the required value of \( p \) and \( q \) are 7 and -18 respectively.

**Question 5.** If \((x^2 - 1)\) is a factor of the polynomial \( px^4 + qx^3 + rx^2 + sx + t \), then prove that 

\[
\begin{align*}
\therefore & \quad p + r + t = q + s = 0
\end{align*}
\]

**Solution:** We have \( f(x) = px^4 + qx^3 + rx^2 + sx + t \)

Since, \((x^2 - 1)\) is a factor of \( f(x) \), \[∵ \quad x^2 - 1 = (x + 1)(x - 1) \]

then \((x + 1)\) and \((x - 1)\) are also factors of \( f(x) \).

\[
\therefore \quad \text{By factor theorem, we have}
\]

\[
\begin{align*}
f(1) &= 0 \quad \text{and} \quad f(-1) = 0
\end{align*}
\]

For \( f(1) = 0 \), \( p(1)^4 + q(1)^3 + r(1)^2 + s(1) + t = 0 \)

\[
\Rightarrow \quad p + q + r + s + t = 0 \quad \cdots(1)
\]

For \( f(-1) = 0 \), \( p(-1)^4 + q(-1)^3 + r(-1)^2 + s(-1) + t = 0 \)

\[
\Rightarrow \quad p - q + r - s + t = 0 \quad \cdots(2)
\]
Adding (1) and (2), we get
\[ p + q + r + s + t = 0 \]
\[ p – q + r – s + t = 0 \]
\[ 2p + 2r + 2t = 0 \]
\[ \Rightarrow 2[p + r + t] = 0 \]
\[ \Rightarrow p + r + t = 0 \] \(\text{(3)}\)

Subtracting (2) from (1), we get
\[ p + q + r + s + t = 0 \]
\[ p – q + r – s + t = 0 \]
\[ (–) (+) (–) (+) (–) (–) \]
\[ 2q + 2s = 0 \]
\[ \Rightarrow 2[q + s] = 0 \]
\[ \Rightarrow q + s = 0 \] \(\text{(4)}\)

From (4) and (3), we get
\[ p + r + t = q + s = 0 \]

**Question 6.** If \( x + y = 12 \) and \( xy = 27 \), find the value of \( x^3 + y^3 \).

**Solution:** Since, \((x + y)^3 = x^3 + y^3 + 3xy (x + y)\)

\[ \Rightarrow (12)^3 = x^3 + y^3 + 3 (27) (12) \]
\[ \Rightarrow x^3 + y^3 = 81(12) - 12^3 \]
\[ = [9^2 - 12^2] \times 12 \]
\[ = [(9 + 12) (9 - 12)] \times 12 \]
\[ = 21 \times 3 \times 12 = 756 \]

**Question 7.** If \( a + b + c = 5 \) and \( ab + bc + ca = 10 \), Then prove that \( a^3 + b^3 + c^3 – 3abc = –25 \).

**Solution:** Since, \( a^3 + b^3 + c^3 – 3abc = (a + b + c) (a^2 + b^2 + c^2 – ab – bc – ca) \)

\[ \Rightarrow a^3 + b^3 + c^3 – 3 abc = (a + b + c) \left[ (a^2 + b^2 + c^2 + 2ab + 2bc + 2ca) – 3ab – 3bc – 3ca \right] \]
\[ = (a + b + c) \left[ (a + b + c)^2 – 3 (ab + bc + ca) \right] \]
\[ = 5[5^2 – 3(10)] \]
\[ = 5[25 – 30] \]
\[ = 5[-5] = –25 \]

**Question 8.** If \( a, b, c \) are all non-zero and \( a + b + c = 0 \), prove that \( \frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3 \).

**Solution:** Since, \( a + b + c = 0 \)

\[ \Rightarrow a^3 + b^3 + c^3 = 3abc \] \(\text{ .... (1)}\)

Now, in \( \frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3 \), we have

\[ \text{LHS} = \frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} \]
\[
\begin{align*}
\frac{a^2 + b^2 + c^2}{abc} &= \left[ \frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} \right] \\
&= \frac{a^3 + b^3 + c^3}{abc}
\end{align*}
\]

[Multiplying and dividing by ‘abc’]

From (1) and (2), we have

\[
LHS = \frac{3abc}{abc} = 3 = RHS
\]

\[
\therefore \quad \frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3
\]

V. HOT QUESTIONS

Q1. If \(x^2 + \frac{1}{x^2} = 18\) then find the value of \(x - \frac{1}{x}\).

\[
\text{Hint: } \left( x - \frac{1}{x} \right)^2 = x^2 + \frac{1}{x^2} - 2\left( x \cdot \frac{1}{x} \right) = 16 - 2 \Rightarrow 18 - 2 = 16 = (4)^2
\]

\[
\Rightarrow \quad \left( x - \frac{1}{x} \right) = \pm 4
\]

Q2. Factorise: \((a - b)^3 + (b - c)^3 + (c - a)^3\)

\[
\text{Hint: Put } a - b = x, b - c = y \text{ and } c - a = z \text{ so that } x + y + z = 0 \Rightarrow x^3 + y^3 + z^3 = 3xyz
\]

\[
\Rightarrow (a - b)^3 + (b - c)^3 + (c - a)^3 = 3(a - b)(b - c)(c - a)
\]

Q3. Factorise: \(14x^6 - 45x^3y^3 - 14y^6\)

\[
\text{Hint: } \text{Let us put } x^3 = a \text{ and } y^3 = b \text{ so that }
\]

\[
14x^6 - 45x^3y^3 - 14y^6 = 14a^2 - 45ab - 14b^2
\]

\[
= 14a^2 - 49ab + 4ab - 14b^2 = (2a - 7b)(7a + 2b)
\]

\[
\Rightarrow 14x^6 - 45x^3y^3 - 14y^6 = (2x^3 - 7y^3)(7x^3 + 2y^3)
\]

Q4. Find the product: \((x - 3y)(x + 3y)(x^2 + 9y^2)\)

\[
\text{Hint: } (x - 3y)(x + 3y) = (x^2 - 9y^2) \text{ and } (x^2 - 9y^2)(x^2 + 9y^2) = (x^4 - 81y^4)
\]

Q5. If \(x^2 - 3x + 2 \text{ divides } x^3 - 6x^2 + ax + b \text{ exactly, then find the value of ‘a’ and ‘b’}\)

\[
\text{Hint: } x^2 - 3x + 2 = (x - 1)(x - 2) \Rightarrow x^3 - 6x^2 + ax + b \text{ is exactly divisible by }
\]

\[
(x - 1) \text{ and } (x - 2) \text{ also i.e. } f(1) = 0 \text{ and } f(2) = 0
\]

\[
\text{Now, } f(1) = 0 \Rightarrow a + b - 5 = 0 \quad \ldots (i)
\]

\[
\text{and } f(2) = 0 \Rightarrow 2a + b - 16 = 0 \quad \ldots (ii)
\]

\[
solving \ (i) \text{ and } (ii) \ a = 11 \text{ and } b = -6
\]
TEST YOUR SKILLS

1. (i) Which of the following is a polynomial?
   \[ \sqrt{3x} - 1; \quad \frac{p^3}{3} - \frac{3}{p}; \quad x^3 + \frac{3x^5}{\sqrt{x}} \]

(ii) Which of the following can be the exponent of the variable?
   a fraction; zero; a non-negative integer

(iii) What is the degree of a quadratic polynomial?

(iv) What is the degree of polynomial \( \sqrt{5} \)?

(v) What is the value of \( 5x - 4x^2 + 2 \) at \( x = -2 \)?

(vi) What is the zero of a zero polynomial?

(vii) If \( p(x) = x + 5 \) then what is the value of \( p(x) + p(-x) \)?

(viii) What is the zero of the polynomial \( 2x^2 + 9x - 5 \)?

(ix) If \( x^{91} + 91 \) is divided by \( x + 1 \) then what is the remainder?

(x) If \( x + 1 \) is a factor of the polynomial \( 3x^2 + kx \) then what is the value of \( k \)?

(xi) Which of the following is a factor of \( (16x^2 - 1) + (1 + 4x) \)?
   \( (4x - 1); (4x + 1); 8x \)

(xii) What is the value of \( 541^2 - 540^2 \)?

(xiii) Which of the following is a factor of \( (x + y)^3 - (x^3 + y^3) \)?
   \( 3xy; -3xy; x^2y^2 \)

(xiv) Write the co-efficient of \( x \) in the expansion of \( (x + 3)^2 \)?

(xv) If \( x + y + z = 0 \) then what is the value of \( x^3 + y^3 + z^3 \)?

(xvi) What is the area of a rectangle whose length and breadth are \( (x + 1) \) and \( (x - 1) \) respectively?

(xvii) If \( x - \frac{1}{x} = 1 \) then what is the value of \( x^2 + \frac{1}{x^2} \)?

(xviii) If \( x + \frac{1}{x} = 1 \) then what is the value of \( x^2 + \frac{1}{x^2} \)?

(xix) What is the zero of \( (x - 1)(x + 1) \)?

(xx) What is the factorisation of \( 1 + x^3 \)?

2. Define the following:
   (i) Algebraic expression
   (ii) Polynomial
   (iii) Degree of an expression
   (iv) Coefficient
   (v) Variable
   (vi) Constant
   (vii) Monomial
   (viii) Binomial
   (ix) Cubic polynomial
   (x) Quadratic polynomial

3. Factorise \( 4x^2 + \frac{1}{4x^2} + 2 - 9y^2 \)

**Hint:** \( 4x^2 + \frac{1}{4x^2} + 2 = \left( 2x + \frac{1}{2x} \right)^2 \) and \( 9y^2 = (3y)^2 \)

\[ \therefore 4x^2 + \frac{1}{4x^2} + 2 - 9y^2 = \left( 2x + \frac{1}{2x} \right)^2 - (3y)^2 \]
4. Factorise $a^6 + 4a^3 - 1$

**Hint:**

$$a^6 + 4a^3 - 1 = (a^2)^3 + (2a^3) - 3(a^2)(a)$$

5. Factorise $(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3$

**Hint:**

$$a^2 - b^2 = A, b^2 - c^2 = B, c^2 - a^2 = C$$

here:

$$A + B + C = a^2 - b^2 + b^2 - c^2 + c^2 - a^2 = 0$$

6. If $x + \frac{1}{x} = 7$, then find the value of $x^4 + \frac{1}{x^4}$.

7. If $x + \frac{1}{x} = 5$, then find the value of $x^3 + \frac{1}{x^3}$.

8. If $a + b + c = 0$, then show that

$$\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3$$

**Hint:**

If $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$

Dividing by $abc$, we have:

$$\frac{a^3}{abc} + \frac{b^3}{abc} + \frac{c^3}{abc} = 3$$

9. Factorise $4x^2 + 9y^2 + 16z^2 + 12xy + 2yz + 16xz$

10. If $x = 7$ and $y = 6$, then find the value of $49x^2 - 84xy + 36y^2$

**ANSWERS**

1. (i) $x^3 + \frac{3x^2}{\sqrt{x}}$

   (ii) non-negative integer

   (iii) 2

   (iv) 0

   (v) 7

   (vi) any real number

   (vii) $10$

   (viii) $\frac{1}{2}$

   (ix) $90$

   (x) $k = 3$

   (xi) $4x + 1$

   (xii) $125$

   (xiii) $3xyz$

   (xiv) $x^2 - 1$

   (xv) $3$

   (xvi) $(1 + x)(1 - x + x^2)$

2. (i) Combination of constants and variables with basic symbols.

   (ii) Algebraic expression having non-negative integral powers of variables.

   (iii) Highest exponent of the variable.

   (iv) In a product of constant and variable, each is the coefficient of the other.

   (v) A symbol which takes on various numerical values.

   (vi) A symbol having fixed numerical value.

   (vii) An expression having only one term.

   (viii) An expression having only two terms.

   (ix) An expression of degree 3.

   (x) An expression of degree 4.

3. $(2x + \frac{1}{2})^2$  

4. $(a^2 + a - 1)(a^4 - a^3 + 2a^2 + a + 1)$

5. $(a^2 - b^3)(b^2 - c^2)(c^2 - a^2)$

6. 2207

7. 110

9. $(2x + 3y + 4z)^2$

10. 169
**HIGHER ORDER THINKING SKILLS (HOTS)**

**Question 1.** If \( p(x) = x^2 - 2 \sqrt{2}x + 1 \), then find \( p(2\sqrt{2}) \).

**Solution:** Since, \( p(x) = x^2 - 2 \sqrt{2}x + 1 \)

Then \( p(2\sqrt{2}) = \left[ (2\sqrt{2}) \right]^2 - [2\sqrt{2}](2\sqrt{2}) + 1 \)

\[ = 4(2) - 4(2) + 1 = 8 - 8 + 1 = 1 \]

**Question 2.** If \( a + b + c = 9 \), and \( ab + bc + ca = 26 \), find \( a^2 + b^2 + c^2 \)

**Solution:**

\[ (a + b + c)^2 = (a^2 + b^2 + c^2) + 2(ab + bc + ca) \]

\[ \Rightarrow \left(9\right)^2 = (a^2 + b^2 + c^2) + 2(26) = (a^2 + b^2 + c^2) + 52 \]

\[ \Rightarrow a^2 + b^2 + c^2 = 9^2 - 52 = 81 - 52 = 29 \]

**Question 3.** Factorise: \( 8p^3 + \frac{12}{5}p^2 + \frac{6}{25}p + \frac{1}{125} \)

**Solution:** \( \therefore 8p^3 = (2p)^3 \), and \( \frac{1}{125} = \left(\frac{1}{5}\right)^3 \)

\[ \therefore 8p^3 + \frac{12}{5}p^2 + \frac{6}{25}p + \frac{1}{125} = (2p)^3 + 3 \left(\frac{1}{5}\right) + 3 \left(\frac{1}{5}\right)^2 + 3 \left(\frac{1}{5}\right)^3 \]

\[ = \left(2p + \frac{1}{5}\right)^3 \]

\[ \therefore a^3 + b^3 + c^3 = 9^2 - 52 = 81 - 52 = 29 \]

**Question 4.** If \( a, b, c \) are all non-zero and \( a + b + c = 0 \), prove that \( \frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3 \)

**Solution:** \( \therefore a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 - 3abc = 0 \)

or \( a^3 + b^3 + c^3 = 3abc \Rightarrow \frac{a^3}{abc} + \frac{b^3}{abc} + \frac{c^3}{abc} = \frac{3abc}{abc} \Rightarrow \frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3 \)

**Question 5.** Prove that \( (a + b + c)^3 - a^3 - b^3 - c^3 = 3(a + b)(b + c)(c + a) \)

**Solution:** L.H.S. = \( (a + b + c)^3 - a^3 - b^3 - c^3 = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \)

\[ \text{using } x^3 - y^3 = (x - y)(x^2 + y^2 + xy) \]

and \( b^3 + c^3 = (b + c)(b^2 - bc + c^2) \)

\[ \text{using } x^3 + y^3 = (x + y)(x^2 + y^2 - xy) \]

From (1), (2) and (3), we get

L.H.S. = \( (a + b + c)(3a^2 + b^2 + c^2 + 3ab + 2bc + 3ca) - (a + b + c)(b^2 + c^2 - bc) \)

\[ = (b + c)(3a^2 + b^2 + c^2 + 3ab + 2bc + 3ca - b^2 - c^2 + bc) \]

\[ = (b + c)(3a^2 + (b^2 - b^2) + (c^2 - c^2) + 3ab + (2bc + bc) + 3ca) \]

\[ = (b + c)(3a^2 + 0 + 0 + 3ab + 3bc + 3ca) \]

\[ = (b + c)(3a^2 + ab + bc + ca) \]

\[ = 3(a + b + c)(c + a) \]

\[ = 3(a + b)(b + c)(c + a) = \text{RHS} \]
VALUE BASED QUESTIONS (VBQs)

Question 1. In a school function, students having 100% attendance are to be honoured. Class teacher of IX A gives the number of eligible students as $\sqrt{9 + 2x} - \sqrt{2x}$ whereas the class teacher of IX B gives the number of such students as $\frac{5}{\sqrt{9 + 2x}}$. If both the above numbers are equal then

(i) Find the number of prize-winners of 100% attendance in each section of class IX.
(ii) Which mathematical concept is used in the above problem?
(iii) By honouring 100% attendance holders, which value is depicted by the school administration?

Solution. (i) Number of eligible students of class IX A

$$= \sqrt{9 + 2x} - \sqrt{2x}$$

Number of eligible students of class IX B

$$= \frac{5}{\sqrt{9 + 2x}}$$

Since, the number of eligible students in both sections are equal.

$$\therefore \sqrt{9 + 2x} - \sqrt{2x} = \frac{5}{\sqrt{9 + 2x}}$$

$$\Rightarrow \sqrt{9 + 2x} \left( \sqrt{9 + 2x} - \sqrt{2x} \right) = 5$$

$$\Rightarrow (\sqrt{9 + 2x})^2 - \sqrt{2x} (9 + 2x) = 5$$

$$\Rightarrow 9 + 2x - 5 = \sqrt{2x} (9 + 2x)$$

$$\Rightarrow 4 + 2x = \sqrt{2x} (9 + 2x)$$

Squaring both sides,

$$(4 + 2x)^2 = \left( \sqrt{2x} (9 + 2x) \right)^2$$

$$\Rightarrow 16 + 4x^2 + 16x = 18x + 4x^2$$

$$\Rightarrow 16 = 2x \Rightarrow x = 8$$

Thus, in each section number of eligible students = 8

(ii) Number system.

(iii) Regularity.

Question 2. Ravita donated ₹ \( \left( x^2 + \frac{1}{x} \right) \) to a blind school. Her friends wanted to know the amount donated by her. She did not disclose the amount but gave a hint that $x + \frac{1}{x} = ₹ 7$.

(i) Find the amount donated by Ravita to blind school.
(ii) Which mathematical concept is involved in this problem?
(iii) By donating an amount to blind school, which value is depicted by Ravita?

Solution. (i) \( \because \)

$$x + \frac{1}{x} = 7 \quad \text{(Given)}$$
\[
\begin{align*}
\therefore \quad \left[ x + \frac{1}{x} \right]^3 &= 7^3 \\
\Rightarrow \quad x^3 + \frac{1}{x^3} + 3x \cdot \frac{1}{x} \left( x + \frac{1}{x} \right) &= 343 \\
\Rightarrow \quad x^3 + \frac{1}{x^3} + 3 \times 1 \times (7) &= 343 \\
\Rightarrow \quad x^3 + \frac{1}{x^3} + 21 &= 343 \\
\Rightarrow \quad x^3 + \frac{1}{x^3} &= 343 - 21 = 322
\end{align*}
\]

Thus, the amount donated by Ravita is Rs 322.

(ii) Polynomials.

(iii) Charity.

**Question 3.** A group of \((a + b)\) teachers, \((a^2 + b^2)\) girls and \((a^3 + b^3)\) boys set out for an ‘Adult Education Mission’. If in the group, there are 10 teachers and 58 girls then:

(i) Find the number of boys.

(ii) Which mathematical concept in used in the above problem?

(iii) By working for 'Adult Education', which value is depicted by the teachers and students?

**Solution.** (i) \(\therefore\) \((a + b)^2 = a^2 + b^2 + 2ab\)

\[
\begin{align*}
10^2 &= 58 + 2ab \\
\therefore \text{Number of teachers} = (a + b) &= 10 \text{ and Number of girls} = (a^2 + b^2) = 58 \\
\Rightarrow \quad 100 &= 58 + 2ab \\
\Rightarrow \quad 2ab &= 100 - 58 = 42 \\
\Rightarrow \quad ab &= \frac{42}{2} = 21
\end{align*}
\]

Now, \(\therefore\) \((a + b)^3 = a^3 + b^3 + 3ab (a + b)\)

\[
\begin{align*}
(10)^3 &= a^3 + b^3 + 3 \times 21 \times 10 \\
\Rightarrow \quad 1000 &= a^3 + b^3 + 630 \\
\Rightarrow \quad a^3 + b^3 &= 1000 - 630 = 370
\end{align*}
\]

\(\therefore\) Number of boys = 370.

(ii) Polynomials.

(iii) Social upliftment.